Generalizing Hook Lengths for Partitions

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National & Kapodistrian University of Athens

CORTIPOM '25

Closing Conference of the Cortipom ANR project

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Proposal ID: 015659 | Acronym: SYMATRAL

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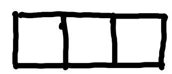

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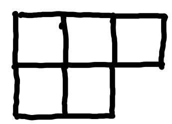

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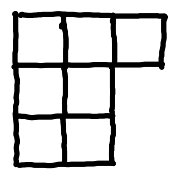

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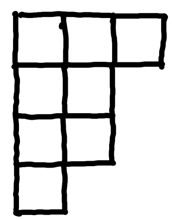

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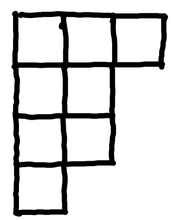

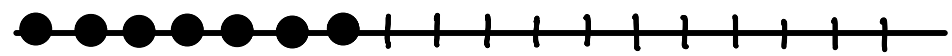

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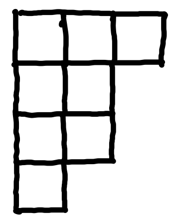

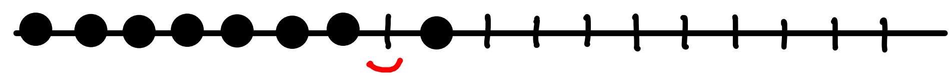

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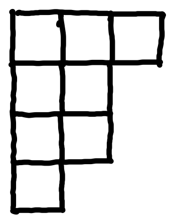

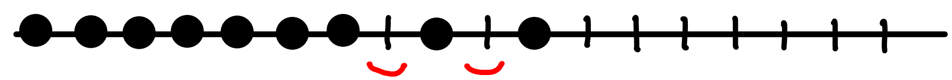

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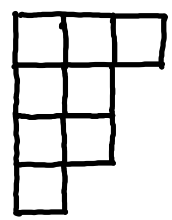

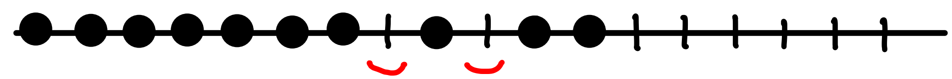

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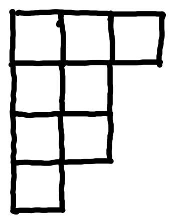

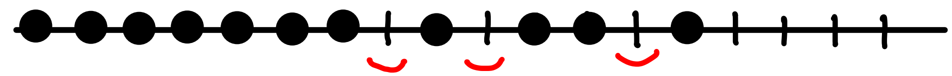

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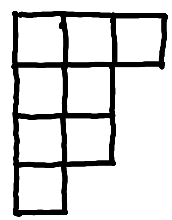

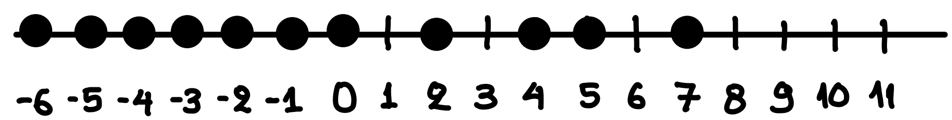

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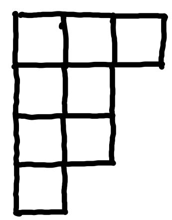

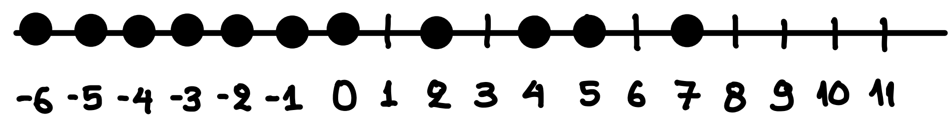

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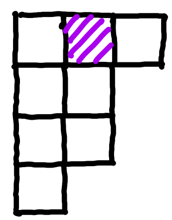

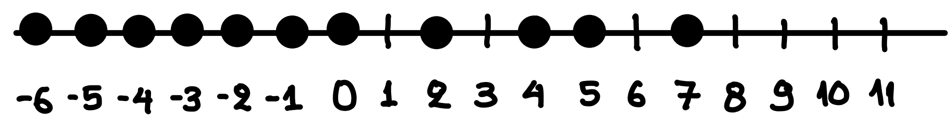

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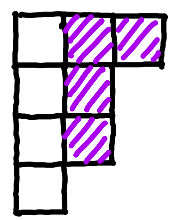

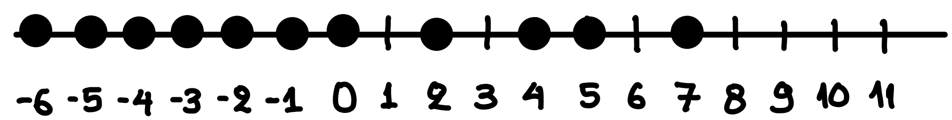

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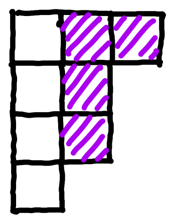

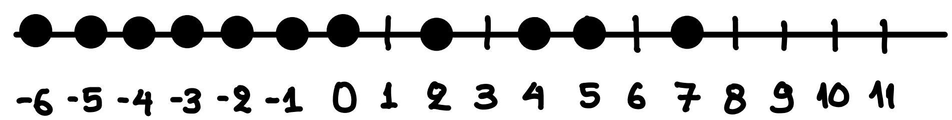

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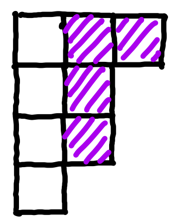

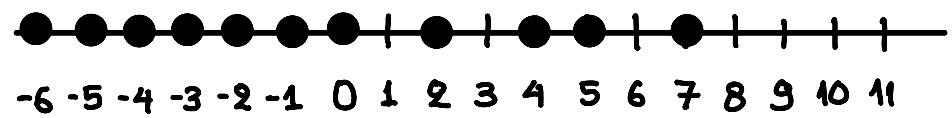

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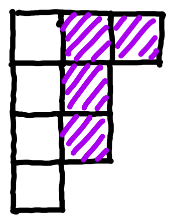

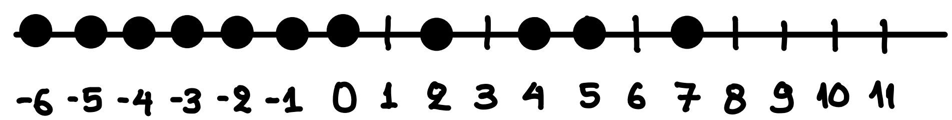

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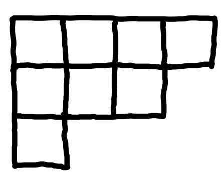

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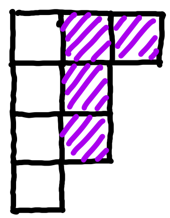

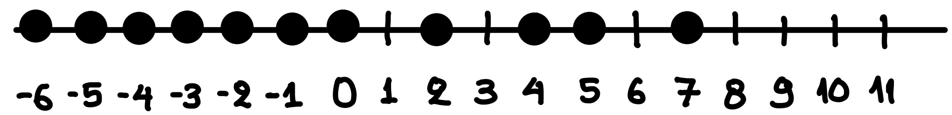

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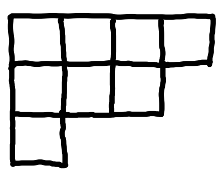

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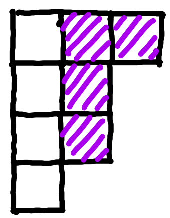

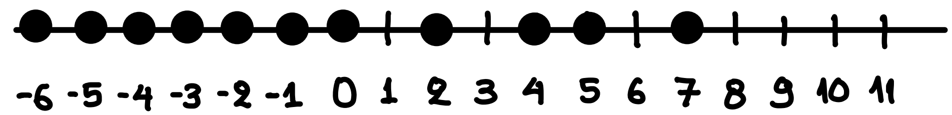

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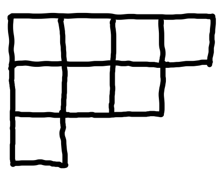

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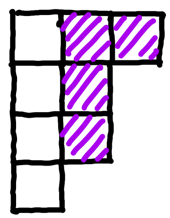

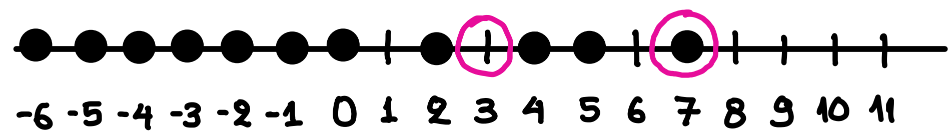

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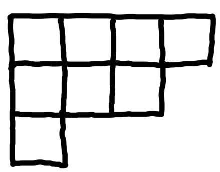

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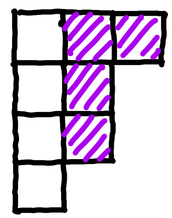

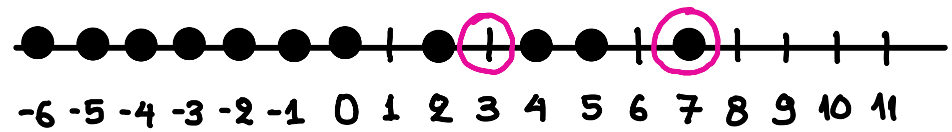

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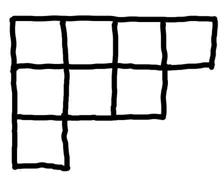

Hook length formula

$$\boxed{\dim V_\lambda = \frac{n!}{\prod_{i,j} h_{i,j}}}$$

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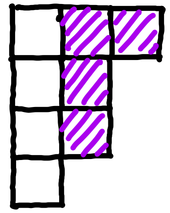
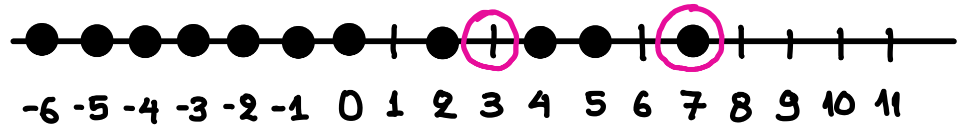


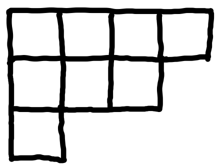


Hook length of $(i,j) \in Y(\lambda)$

$$\boxed{h_{i,j} = \lambda_i - i + \lambda'_j - j + 1}$$

Ex. $h_{1,2} = 3-1+3-2+1 = 7-3 = 4$

$\lambda' = (4, 3, 1)$



Hook length formula

$$\boxed{\dim V_\lambda = \frac{n!}{\prod_{i,j} h_{i,j}}}$$

Ex. Hook lengths are: $7-1,\ 7-3,\ 7-6,\ 5-1,\ 5-3,\ 4-1,\ 4-3,\ 2-1$

$$\dim V_\lambda = \frac{8!}{6\cdot 4\cdot 1\cdot 4\cdot 2\cdot 3\cdot 1\cdot 1} = 70$$

$\mathrm{Irr}(kS_n) \longleftrightarrow \{\text{Partitions of } n\}$

when $\mathrm{char}\, k \nmid n!$

$$\mathrm{Irr}(kS_n) \longleftrightarrow \{\text{p-regular partitions of } n\}$$

when $\operatorname{char} k = p \mid n!$

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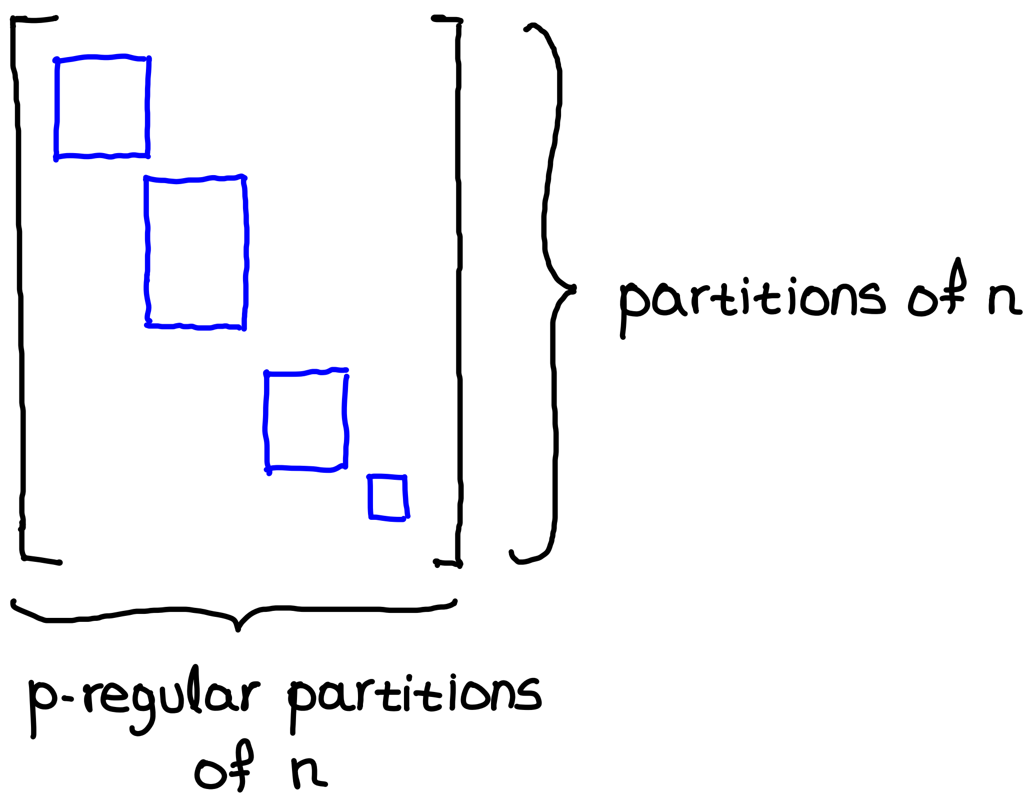

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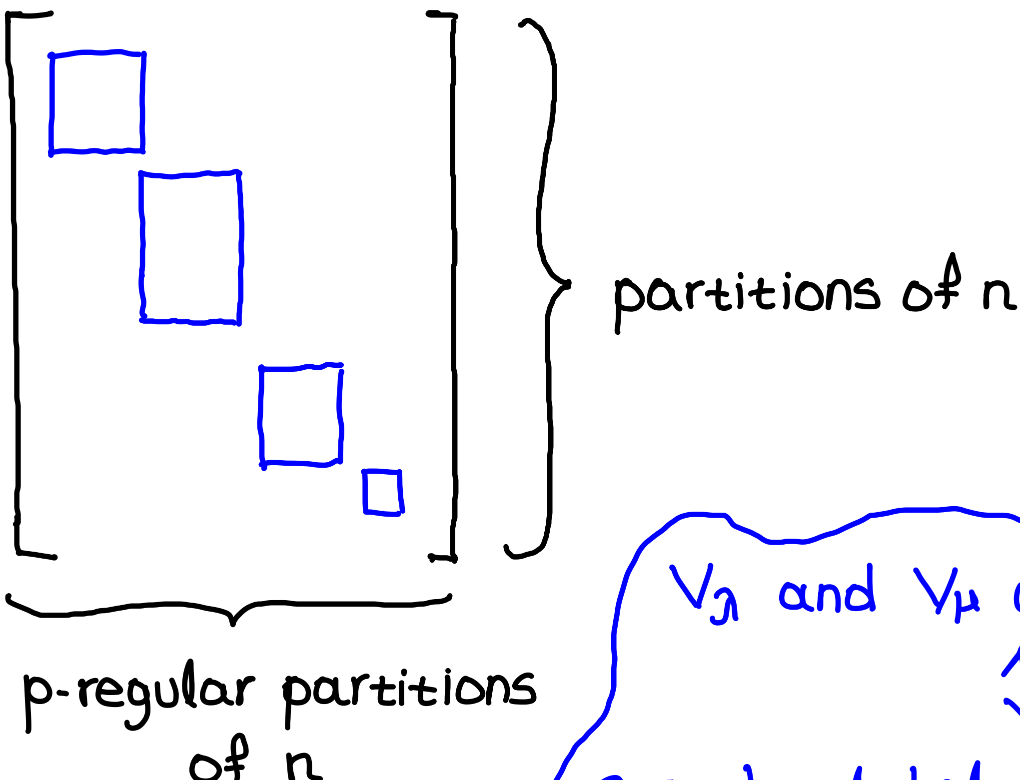
$D =$ 

V_λ and V_μ are in the same block

$\Updownarrow$

λ and μ label rows in the same block of D

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<u>Ex.</u> $\lambda = (3,2,2,1)$ is p-regular for $p > 2$

<u>BLOCKS</u>: V_λ and V_μ are in the same block B

$$\Updownarrow$$

λ and μ have the same p-core

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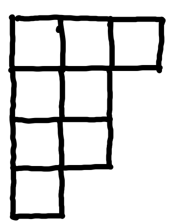

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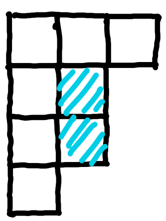

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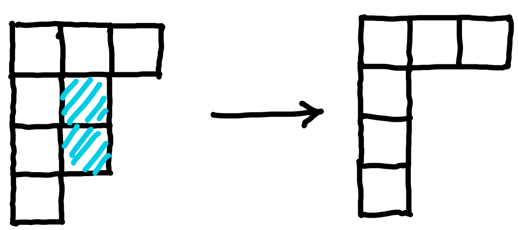

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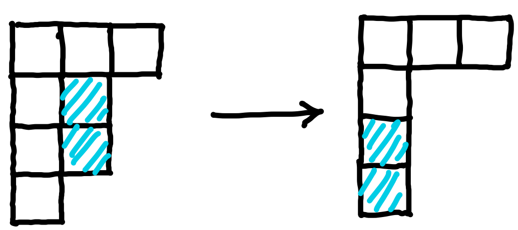

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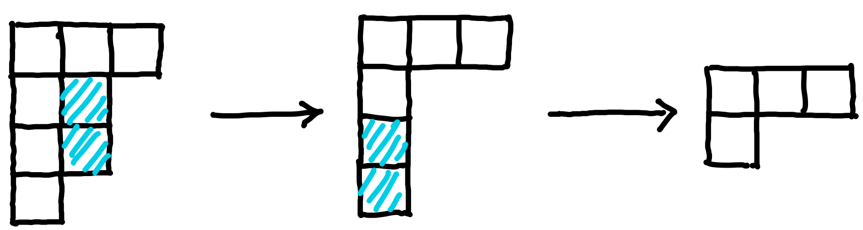

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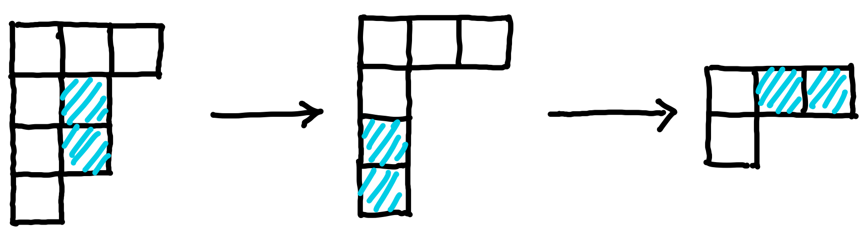

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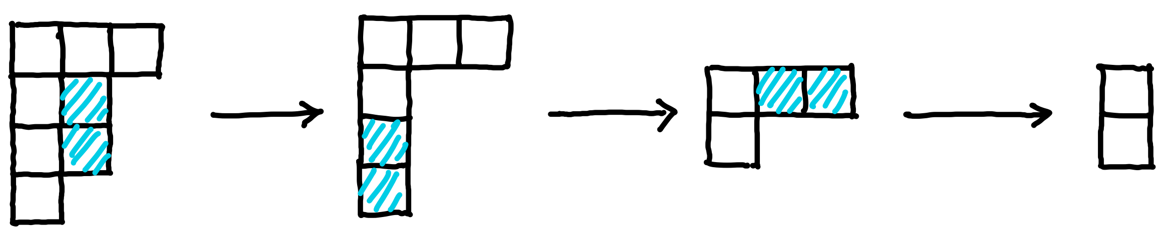

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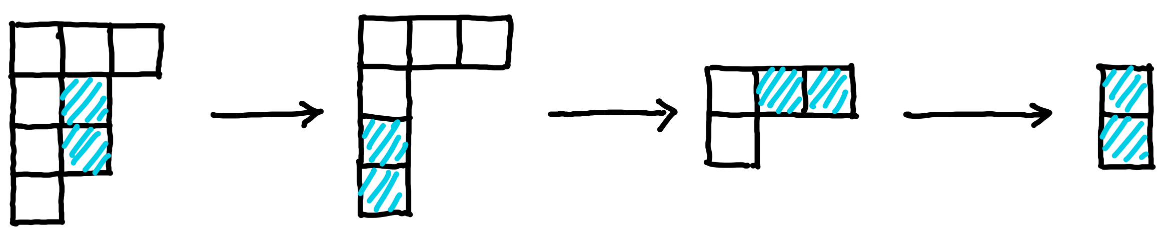

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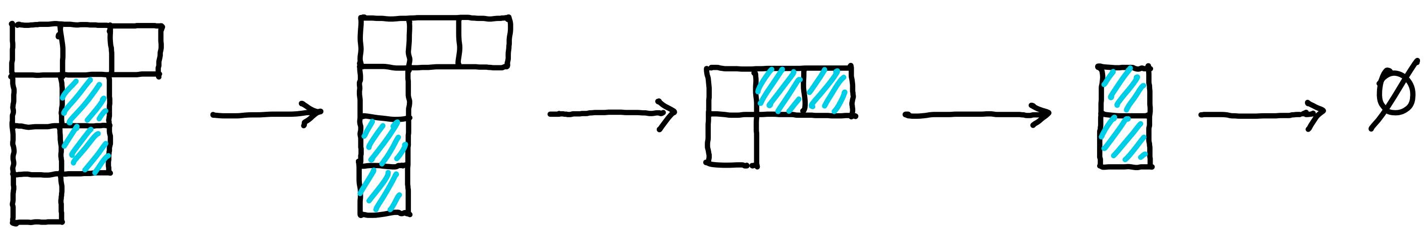

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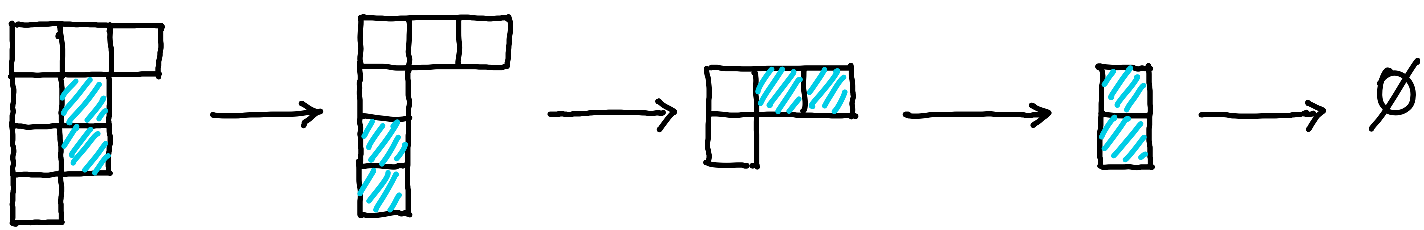

Defect of λ: $d_\lambda = v_p\left(\prod_{i,j} h_{i,j}\right)$

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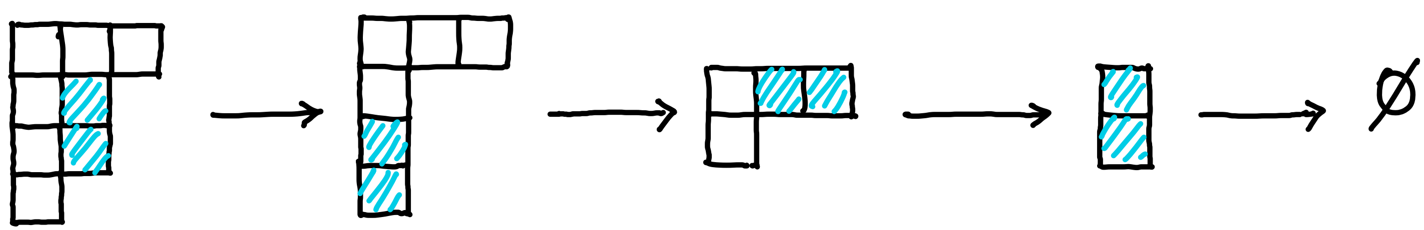

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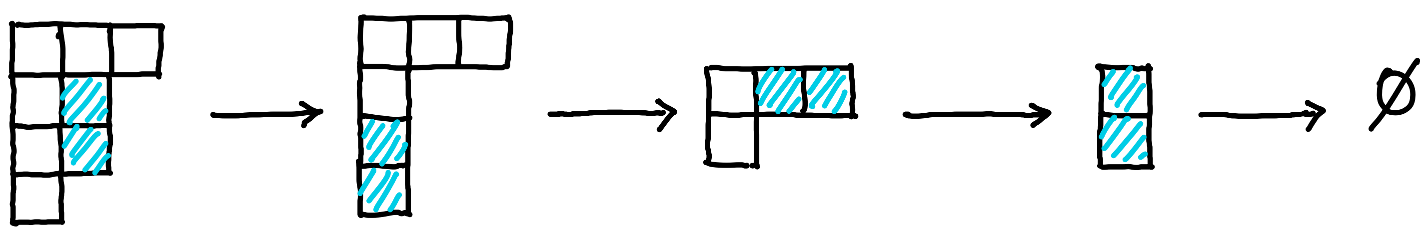

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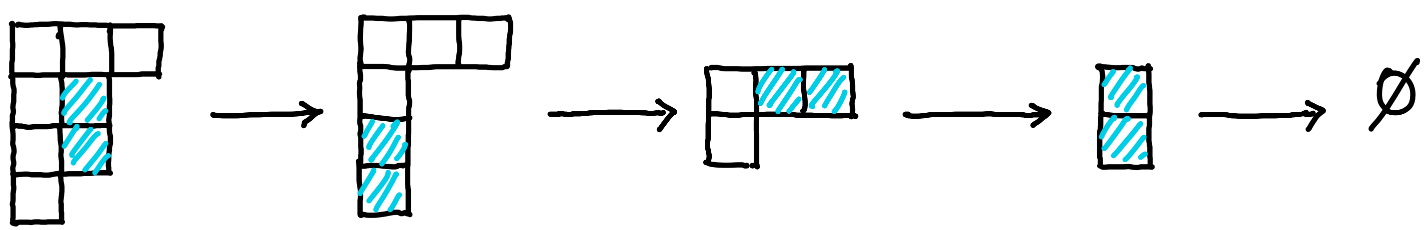

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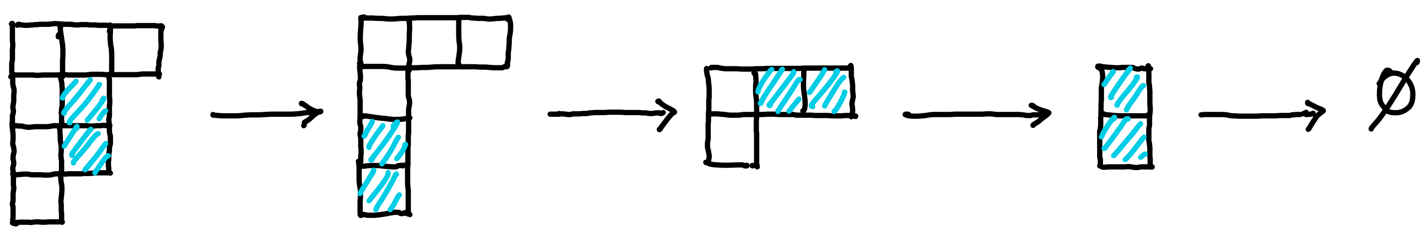

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when $\mathrm{char}\, k = p \mid n!$

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Block Invariant

Ex. $\lambda = (3,2,2,1)$ $\quad HL(\lambda) = \{\{6,4,4,3,2,2,1,1,1\}\}$

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$d_\lambda = v_2(2^7 \cdot 3^2) = 7$

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<u>Def^n</u>: $\prod_{i,j} h_{i,j} = \dfrac{|S_n|}{\dim V_\lambda}$ is the Schur element s_λ of λ

$$\mathbb{C}S_n = \left\langle s_1, \dots, s_{n-1} \;\middle|\; \begin{array}{l} s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \\ s_i s_j = s_j s_i \quad \text{if } |i-j| > 1 \\ s_i^2 = 1 \end{array} \right\rangle$$

$$\mathbb{C}(q)\mathcal{H}_n(q) = \left\langle s_1, \dots, s_{n-1} \;\middle|\; \begin{array}{l} s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \\ s_i s_j = s_j s_i \quad \text{if } |i-j| > 1 \\ s_i^2 = (q-1)s_i + q \end{array} \right\rangle$$

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$$\mathrm{Irr}(\mathbb{C}(q)\mathcal{H}_n(q)) \longleftrightarrow \mathrm{Irr}(\mathbb{C}S_n)$$

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$$\mathrm{Irr}(\mathbb{C}(q)\mathcal{H}_n(q)) \longleftrightarrow \mathrm{Irr}(\mathbb{C}S_n)$$

when q is not a root of unity of order $\leq n$

$$\mathbb{C}(q)\mathcal{H}_n(q) = \left\langle s_1,\dots,s_{n-1} \;\middle|\; \begin{array}{l} s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \\ s_i s_j = s_j s_i \quad \text{if } |i-j|>1 \\ s_i^2 = (q-1)s_i + q \end{array} \right\rangle$$

$$s_\lambda = \frac{q^{-a_\lambda}}{(q-1)^n} \prod_{i,j} \left(q^{h_{i,j}} - 1\right)$$

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<u>Thm</u> (Geck '92) : The defect is a block invariant

$$\mathbb{C}(q)\mathcal{H}_n(q) = \left\langle s_1, \dots, s_{n-1} \;\middle|\; \begin{array}{l} s_i s_{i+1} s_i = s_{i+1} s_i s_{i+1} \\ s_i s_j = s_j s_i \quad \text{if } |i-j|>1 \\ s_i^2 = (q-1)s_i + q \end{array} \right\rangle$$

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<u>Thm</u> (Geck '92) : The defect is a block invariant

<u>Conj</u> (Geck '92) : This holds for all real reflection groups

Def^n $\lambda = (\lambda^{(1)}, \lambda^{(2)}, \dots, \lambda^{(d)})$ d-partition of n if $\forall\ i = 1, \dots, d$
$\lambda^{(i)}$ is a partition and $\sum |\lambda^{(i)}| = n$

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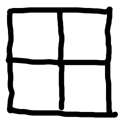
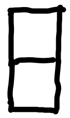
__Ex.__ ( , ) is a 2-partition of 6

Defⁿ $\lambda = (\lambda^{(1)}, \lambda^{(2)}, \ldots, \lambda^{(d)})$ d-partition of n if $\forall\, i = 1, \ldots, d$
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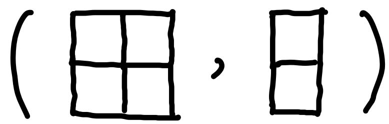
Ex.  is a 2-partition of 6

$\{$ d-partitions of $n \} \longleftrightarrow \mathrm{Irr}(\mathbb{C}G(d,1,n))$

where $G(d,1,n) = (\mathbb{Z}/d\mathbb{Z})^n \rtimes S_n$

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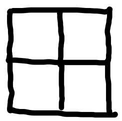
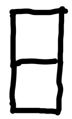
<u>Ex.</u> ( , ) is a 2-partition of 6

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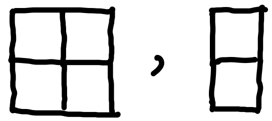
Ex. () is a 2-partition of 6

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Def^n (C. - 07/01/11)

$(i,j) \in Y(\lambda^{(a)})$

$$\boxed{h^{a,b}_{i,j} = \lambda^{(a)}_i - i + \lambda^{(b)\prime}_j - j + 1}$$

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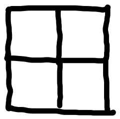
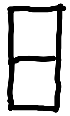
Ex. ( , ) is a 2-partition of 6

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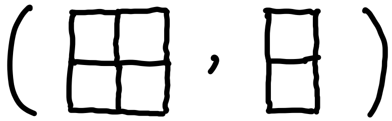
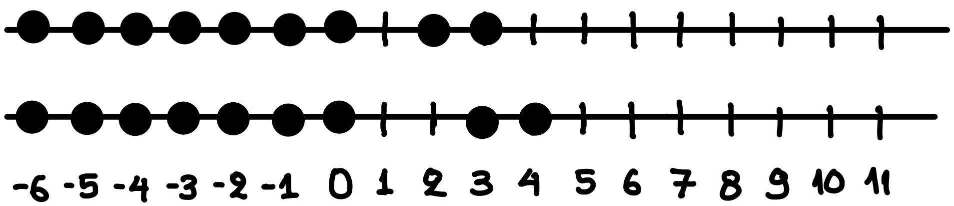
Thm (C.)

$$S_\lambda = \frac{q^{-a_\lambda}}{(q-1)^n} \prod_{a,b} \prod_{i,j} \left(q^{h_{i,j}^{a,b}} - 1\right)$$

j.w. N. Jacon

Ex. 



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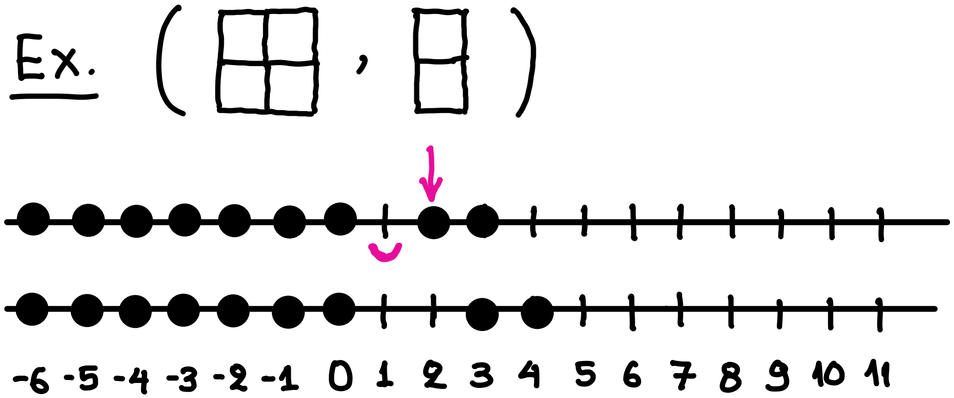

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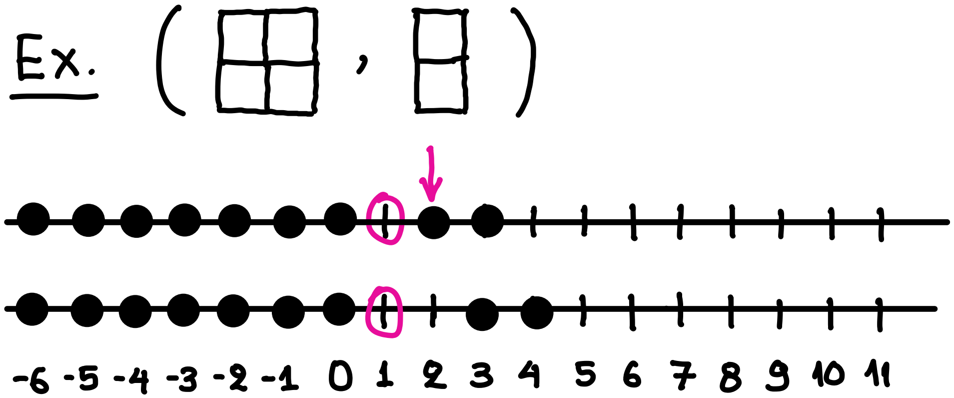
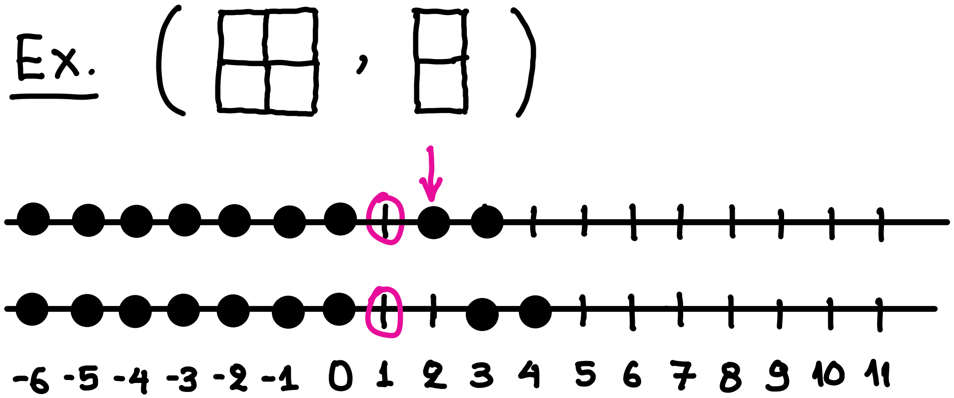
Ex.

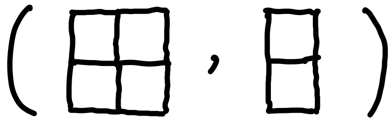


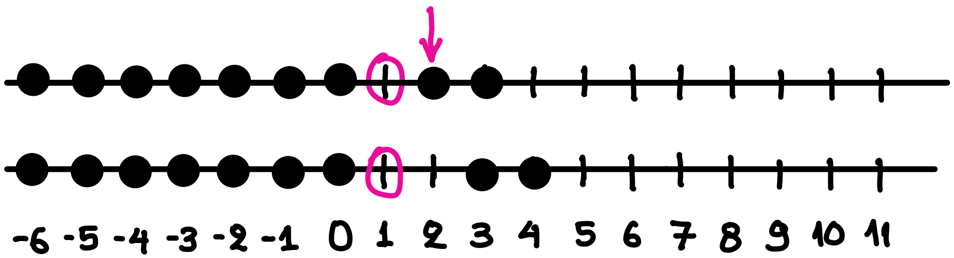
j.w. N. Jacon

Ex. 

$HL^{CJ}(\lambda) = \{\{\ 2-1\ ,\ 2-1$

j.w. N. Jacon

Ex. 



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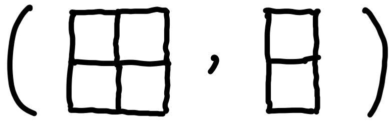
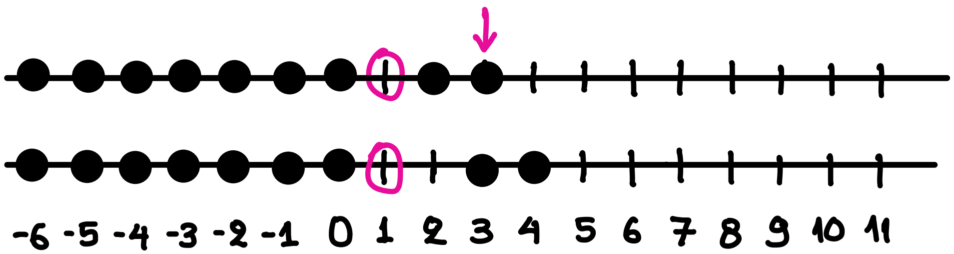
Ex.





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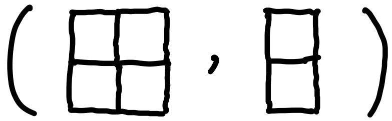
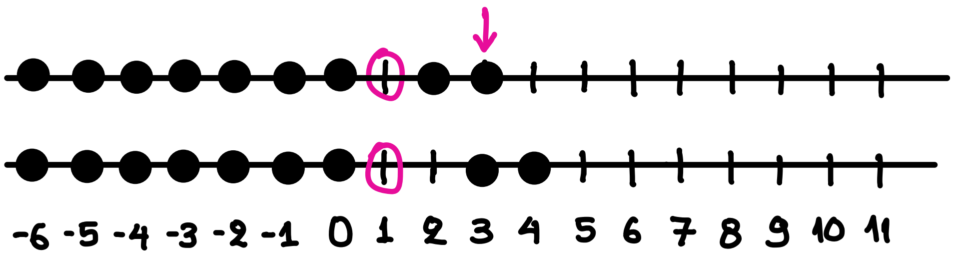
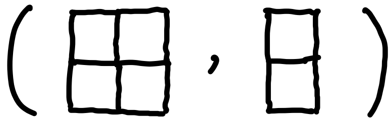
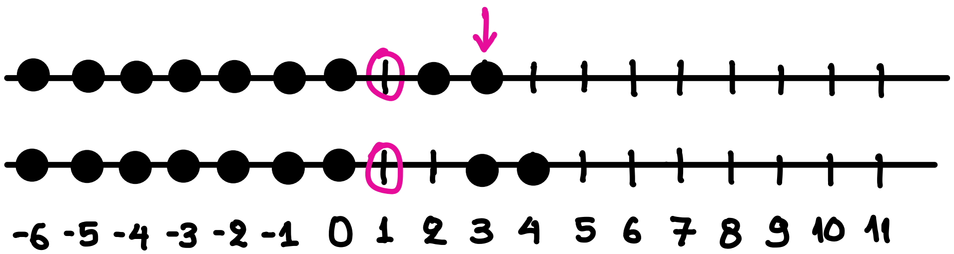
Ex. 



$HL^{CJ}(\boldsymbol{\lambda}) = \{\{ 1, 1, 3-1, 3-1$

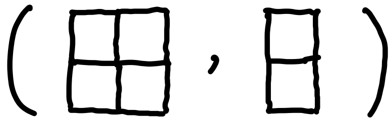
j.w. N. Jacon

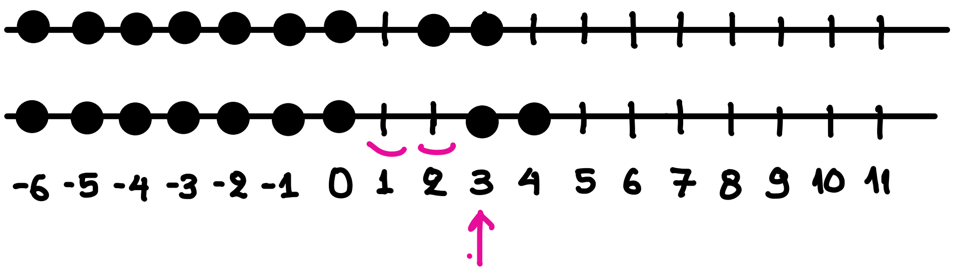
Ex. 



$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2$

j.w. N. Jacon

Ex. 



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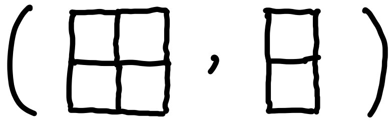

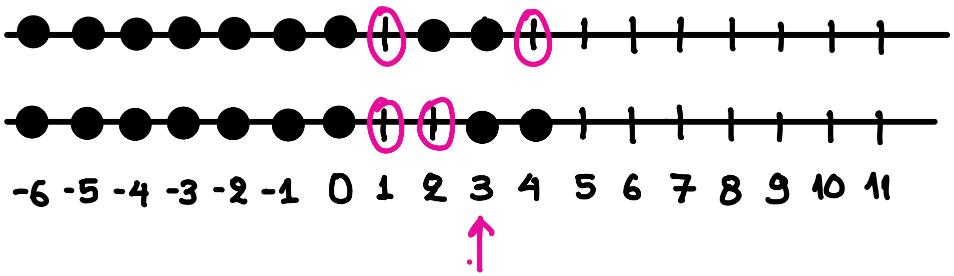

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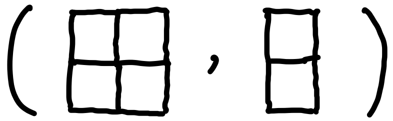
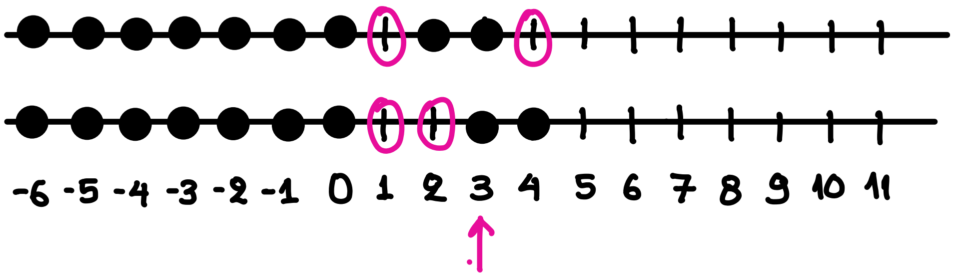
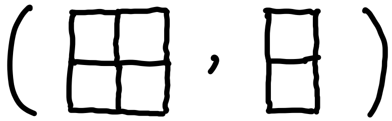
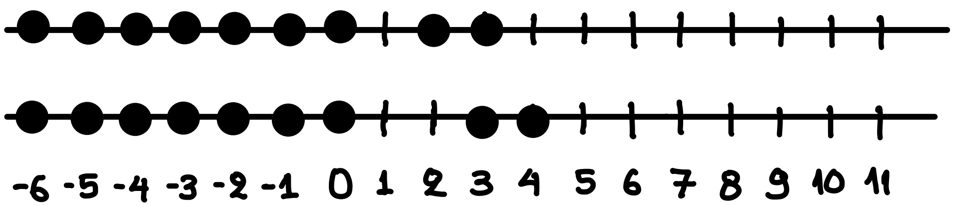
Ex. 



$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2, 3-1, 3-2, 3-1, 3-4$

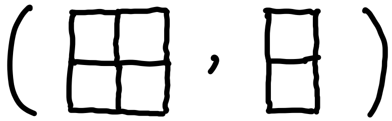
j.w. N. Jacon

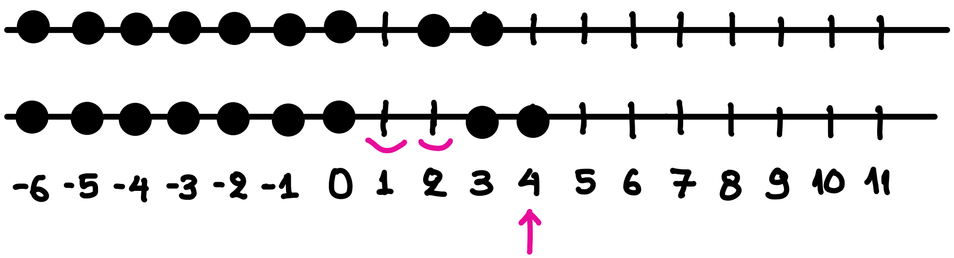
Ex. 



$HL^{cJ}(\lambda) = \{\{ 1, 1, 2, 2, \; 2, 1, 2, -1$

j.w. N. Jacon

Ex. 



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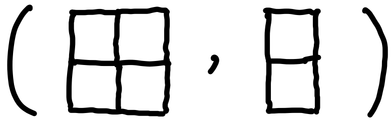
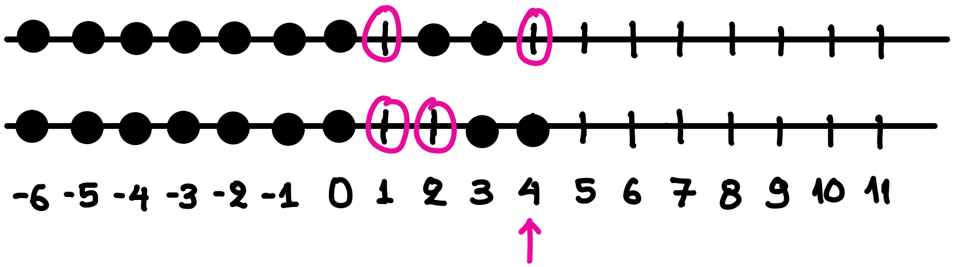
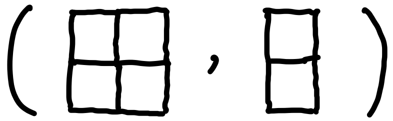
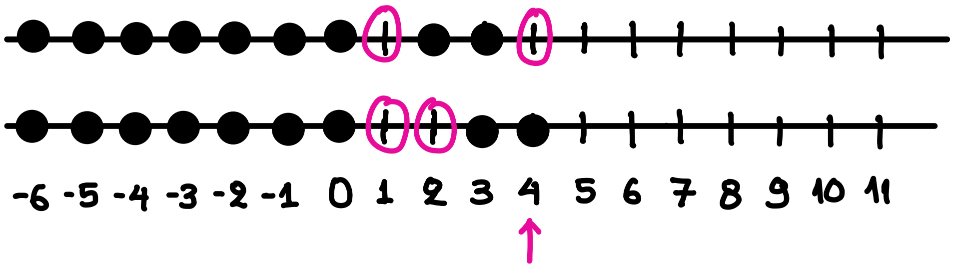
Ex. 



$HL^{CJ}(\boldsymbol{\lambda}) = \{\{ 1, 1, 2, 2, 2, 1, 2, -1$

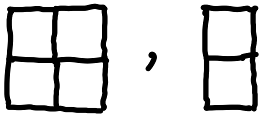
j.w. N. Jacon

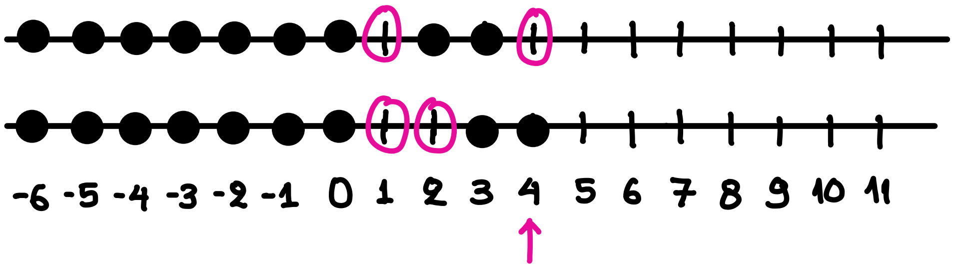
Ex. 



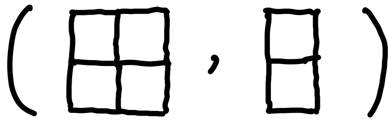
$HL^{CJ}(\lambda) = \{\{\ 1, 1, 2, 2,\ 2, 1, 2, -1, 4-1, 4-2, 4-1, 4-4$

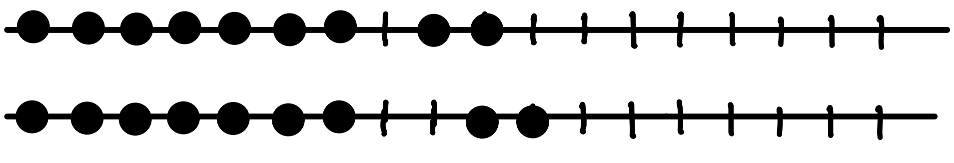
j.w. N. Jacon

Ex. ()



$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$

Ex. 



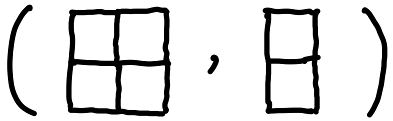
j.w. N. Jacon

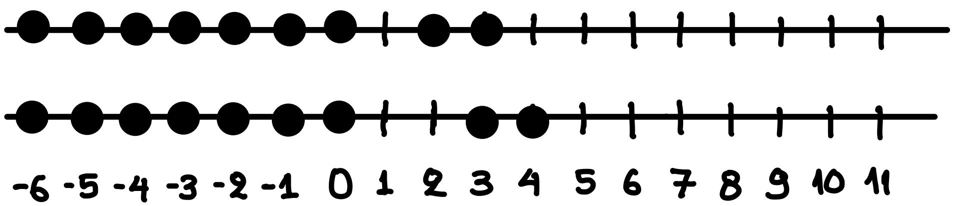
$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$

Let q be a root of unity of order $2 \leq e \leq n$

$$d_\lambda = v_{\Phi_e}(s_\lambda) = \#\{(i,j) : e \mid h_{i,j}^{a,b}\}$$

j.w. N. Jacon

Ex. 



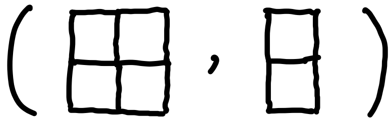
$$HL^{CJ}(\lambda) = \{\{1,1,2,2,\ 2,1,2,-1,3,2,3,0\}\}$$

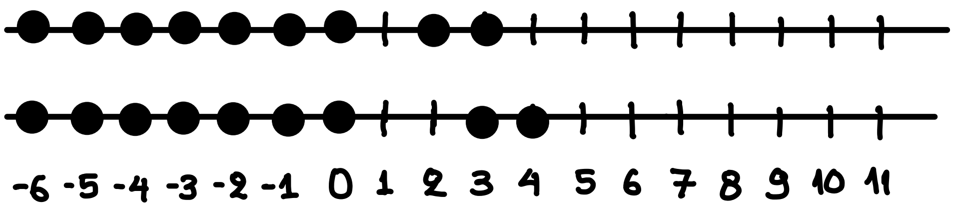
Let q be a root of unity of order $2 \leq e \leq n$

$$d_\lambda = \nu_{\Phi_e}(s_\lambda) = \#\{(i,j) : e \mid h_{i,j}^{a,b}\}$$

Thm (C.-Jacon'23) : The defect is a block invariant

j.w. N. Jacon

Ex. 



$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$

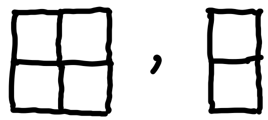
Let q be a root of unity of order $2 \leq e \leq n$

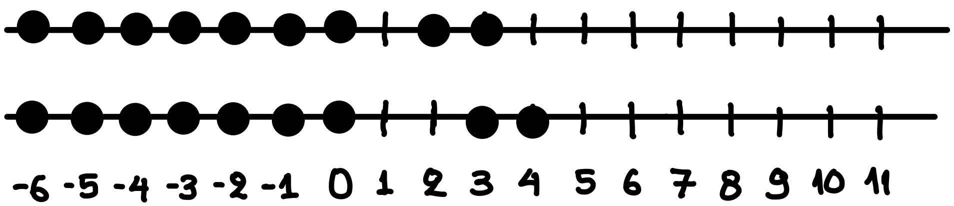
$$d_\lambda = \nu_{\Phi_e}(s_\lambda) = \#\{(i,j) : e \mid h_{i,j}^{a,b}\}$$

Thm (C.-Jacon '23) : The defect is a block invariant

Conj (C.-Jacon '23) : This holds for all complex reflection groups

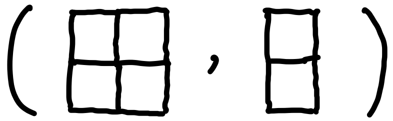
j.w. N. Jacon

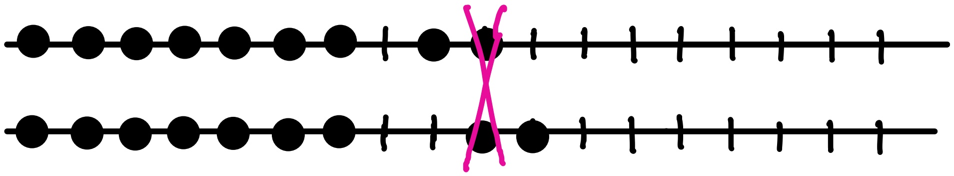
Ex. ()



$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$

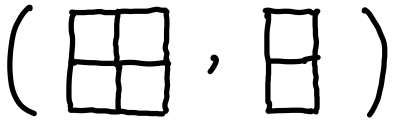
j.w. N. Jacon

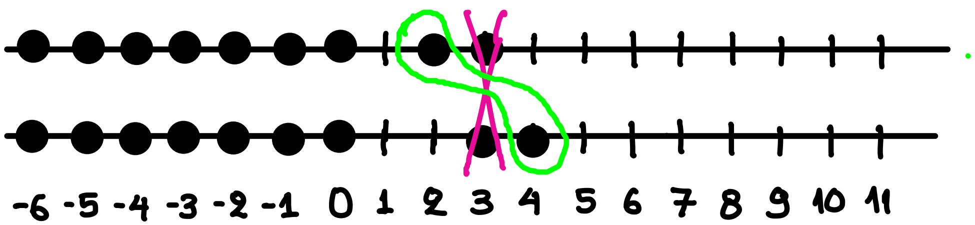
Ex. 



$$HL^{CJ}(\lambda) = \{\{1,1,2,2,\ 2,1,2,-1,3,2,3,0\}\}$$

j.w. N. Jacon

Ex. 



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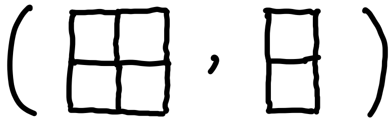
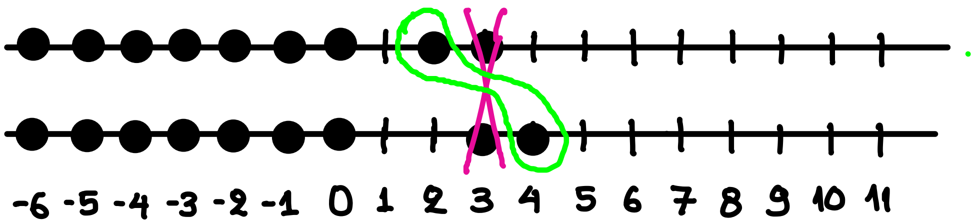
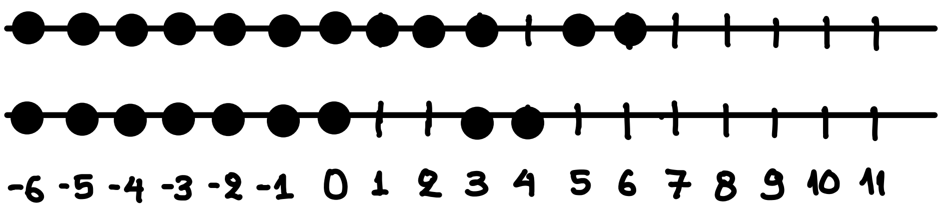
j.w. N. Jacon

Ex. 



$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$



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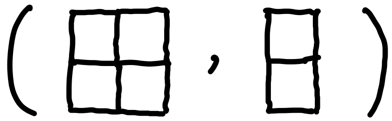
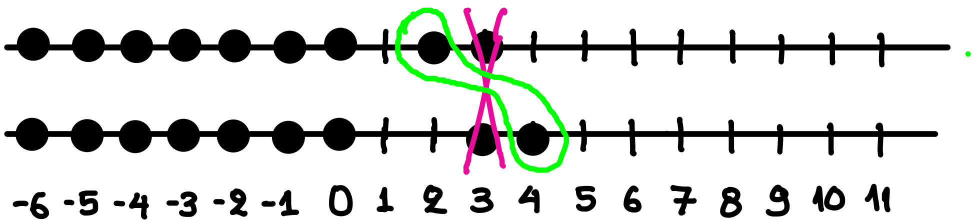
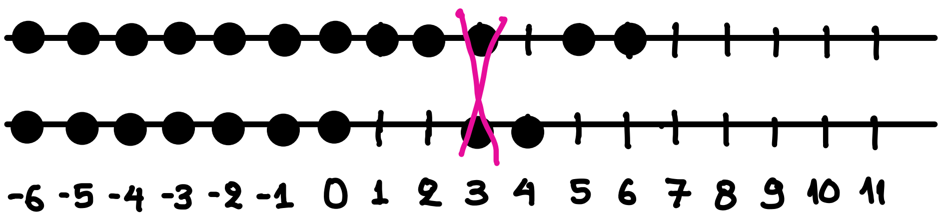
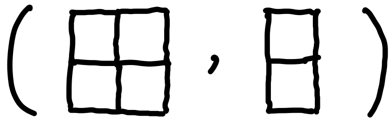
Ex. 

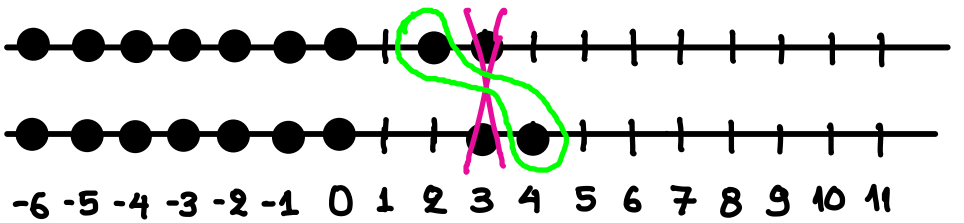


$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$

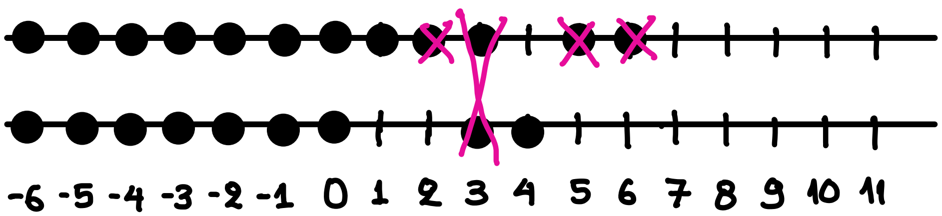


j.w. N. Jacon

Ex. 



$$HL^{CJ}(\lambda) = \{\{1,1,2,2,\ 2,1,2,-1,3,2,3,0\}\}$$



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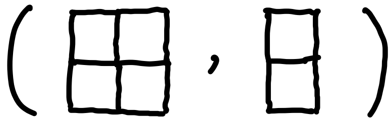
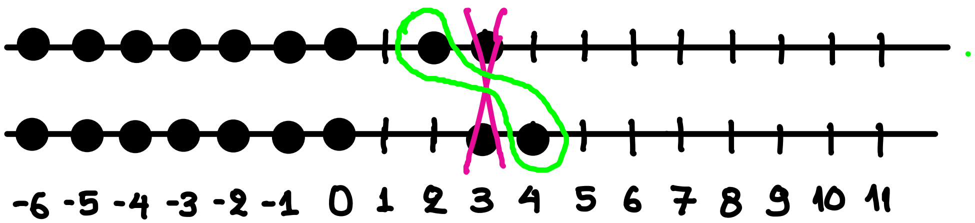
Ex. 



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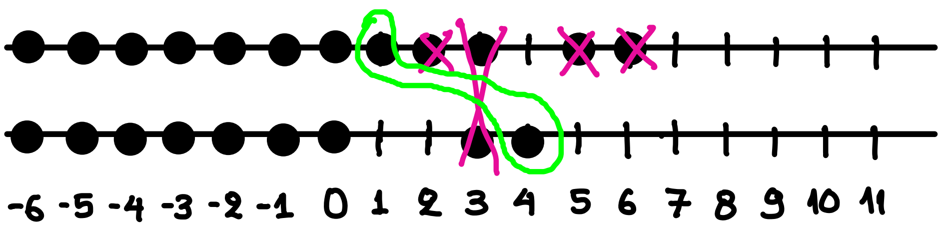
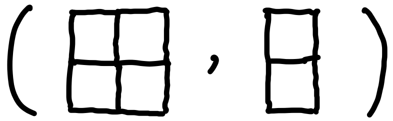
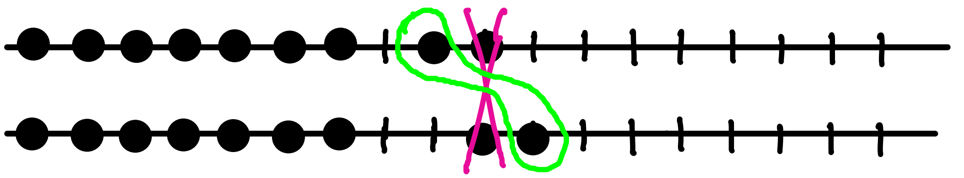
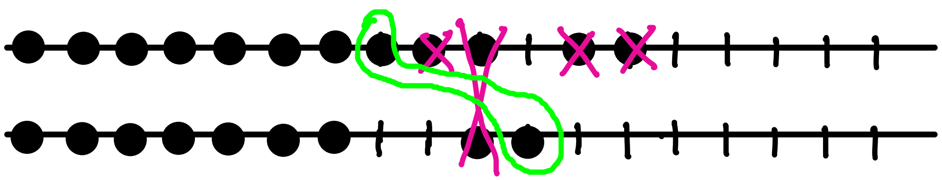


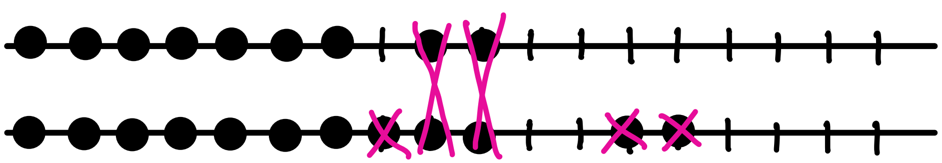
j.w. N. Jacon

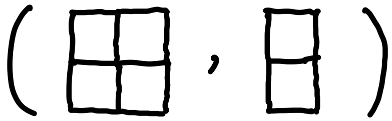
Ex. 



$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$





Ex. 

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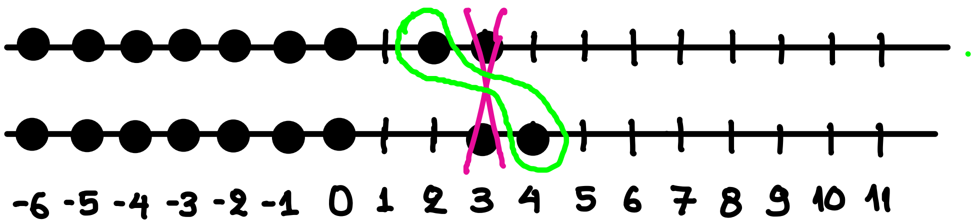

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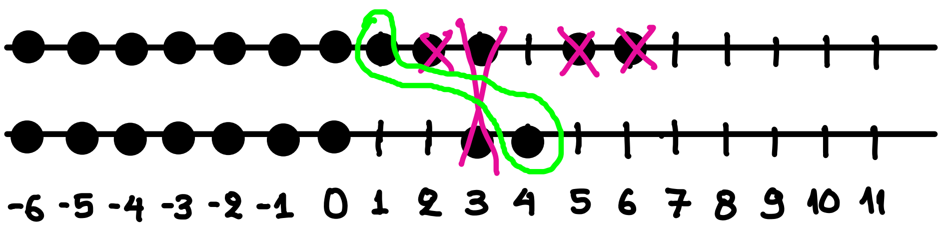
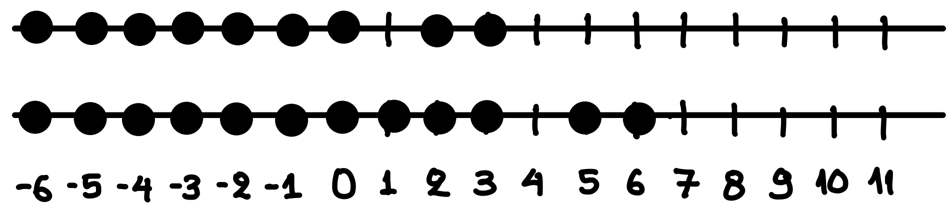
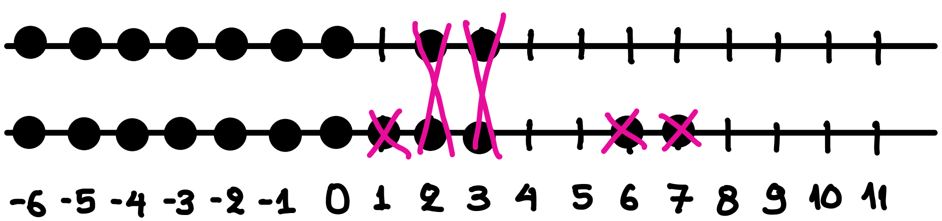
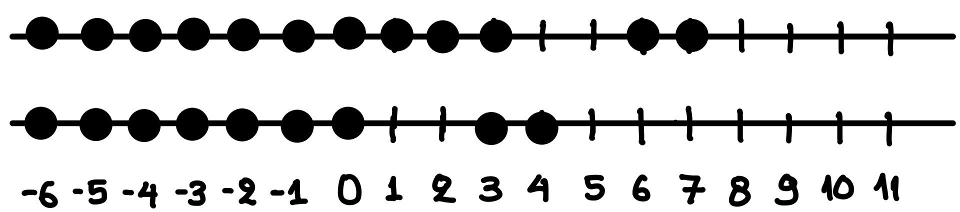
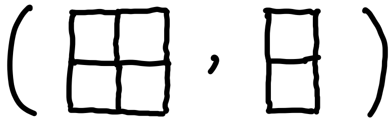


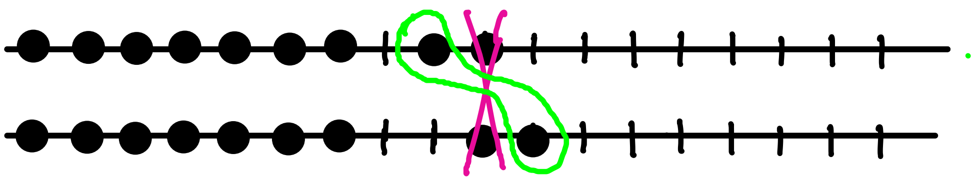




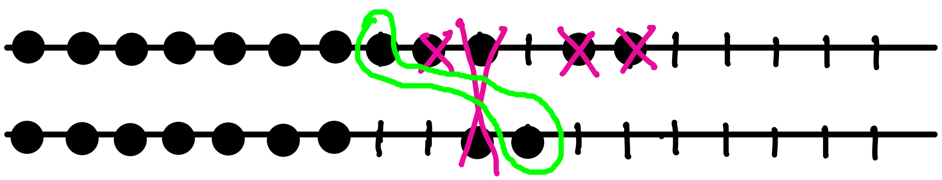


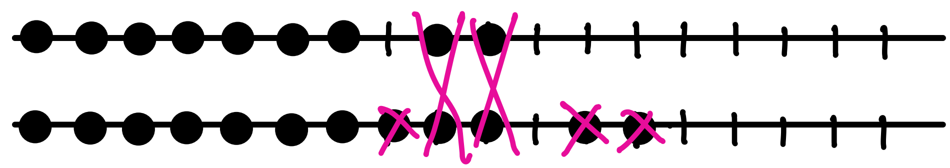
Ex. 

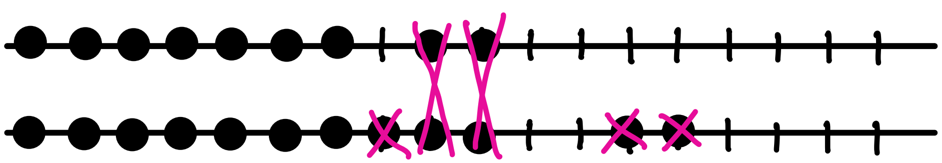
j.w. N. Jacon

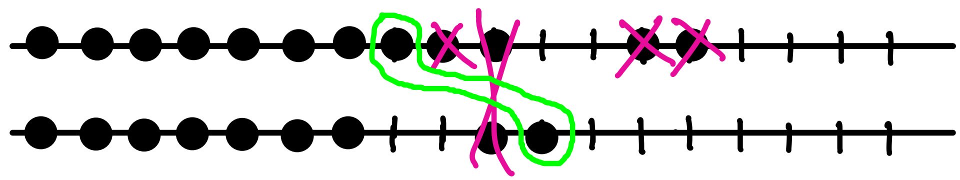


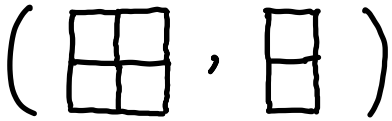
$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$

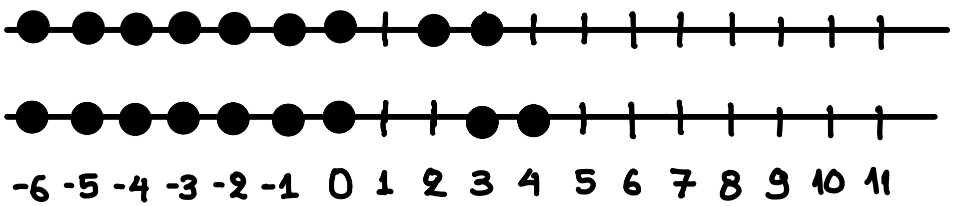








Ex. 

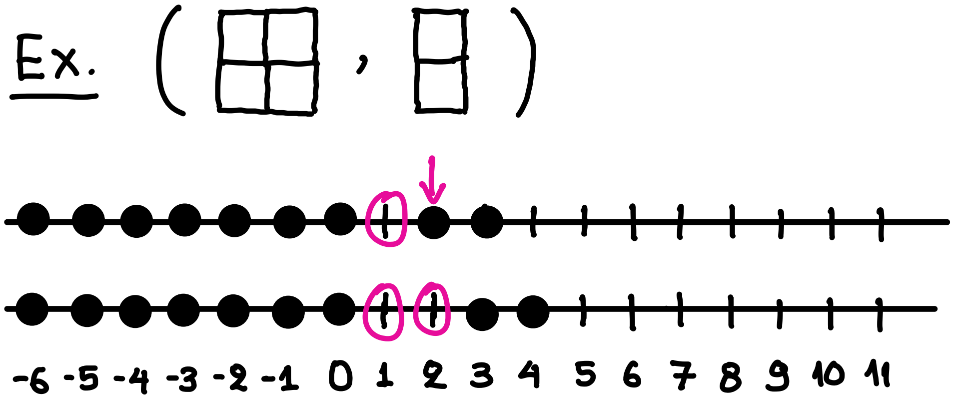


$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$

Defⁿ (Bessenrodt - Gramain - Olsson, 26/01/11)

$HL^{BGO}(\lambda) = \{\{$

Ex.

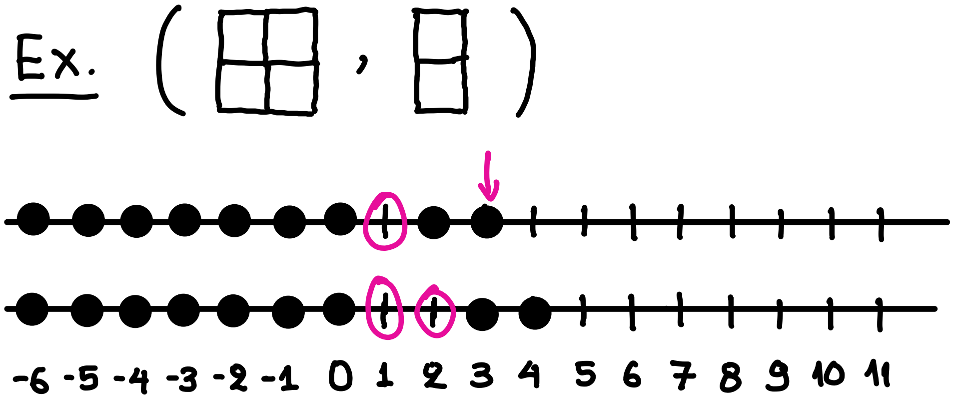


$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$

Defn (Bessenrodt–Gramain–Olsson, 26/01/11)

$$HL^{BGO}(\lambda) = \{\{1, 1, 0,$$

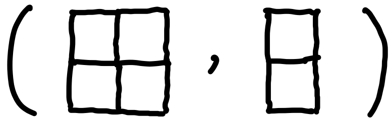
Ex.

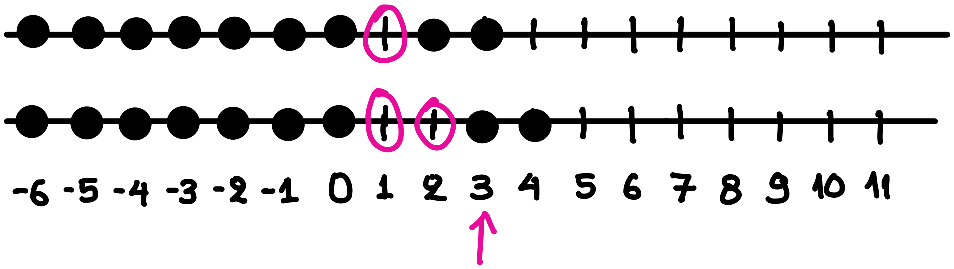


$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0 \}\}$

Def^n (Bessenrodt-Gramain-Olsson, 26/01/11)

$HL^{BGO}(\lambda) = \{\{ 1, 1, 0, 2, 2, 1$

Ex. 



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Def^n (Bessenrodt - Gramain - Olsson, 26/01/11)

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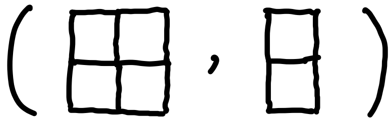
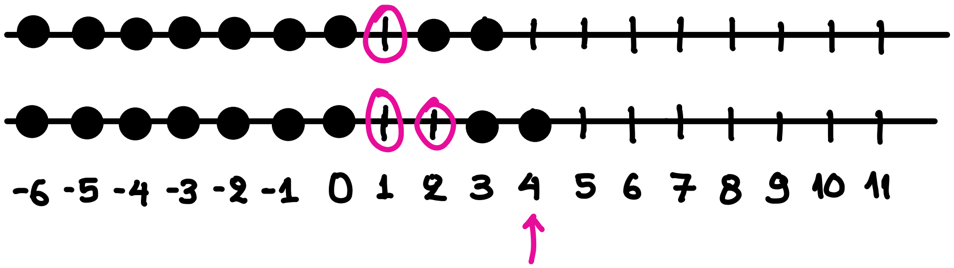
Ex. 

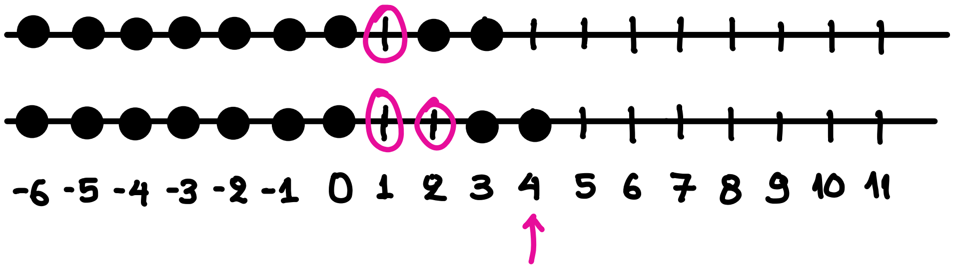


$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$

Def^n (Bessenrodt - Gramain - Olsson, 26/01/11)

$$HL^{BGO}(\lambda) = \{\{1, 1, 0, 2, 2, 1, 2, 2, 1, 3, 3, 2\}\}$$

Ex. $\left((2,2), (1,1) \right)$



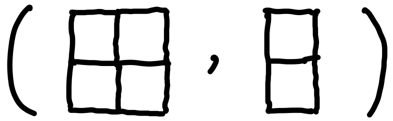
$$HL^{CJ}(\boldsymbol{\lambda}) = \{\{1,1,2,2,\ 2,1,2,-1,3,2,3,0\}\}$$

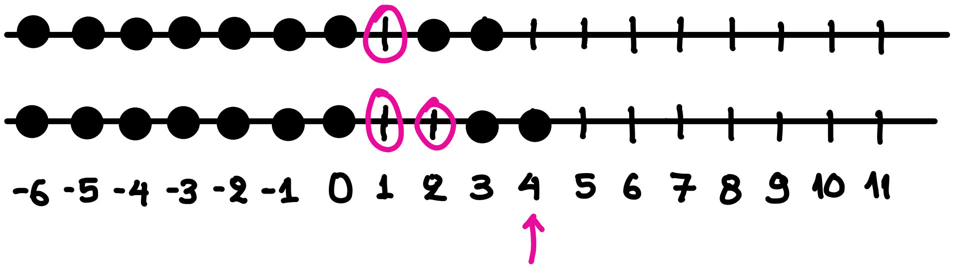
Def^n (Bessenrodt – Gramain – Olsson, 26/01/11)

$$HL^{BGO}(\boldsymbol{\lambda}) = \{\{1,1,0,2,\ 2,1,2,2,1,3,3,2\}\}$$

Thm (C. – Gramain – Jacon, '25)

There exists a bijection $HL^{CJ}(\boldsymbol{\lambda}) \longrightarrow HL^{BGO}(\boldsymbol{\lambda})$, $h \longmapsto |h|$

Ex. 



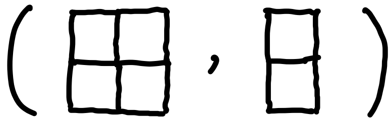
$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0 \}\}$

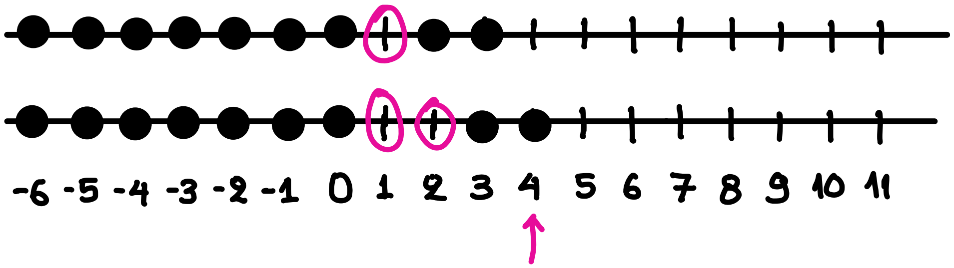
Defⁿ (Bessenrodt – Gramain – Olsson, 26/01/11)

$HL^{BGO}(\lambda) = \{\{ 1, 1, 0, 2, 2, 1, 2, 2, 1, 3, 3, 2 \}\}$

Thm (C.-Gramain-Jacon, '25)

There exists a bijection $HL^{CJ}(\lambda) \longrightarrow HL^{BGO}(\lambda)$, $h \longmapsto |h|$ (when the charges are equal)

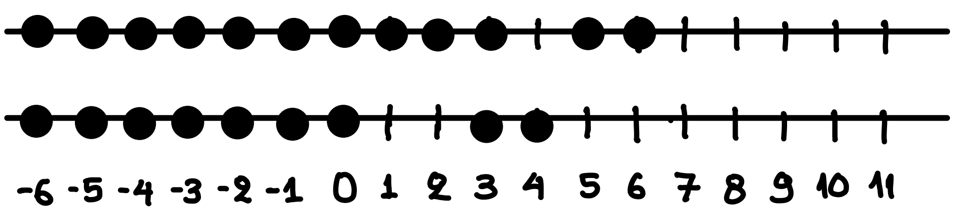
Ex. 



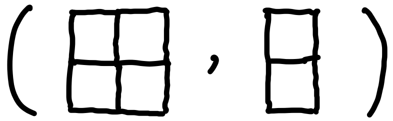
$$HL^{CJ}(\lambda) = \{\{1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0\}\}$$

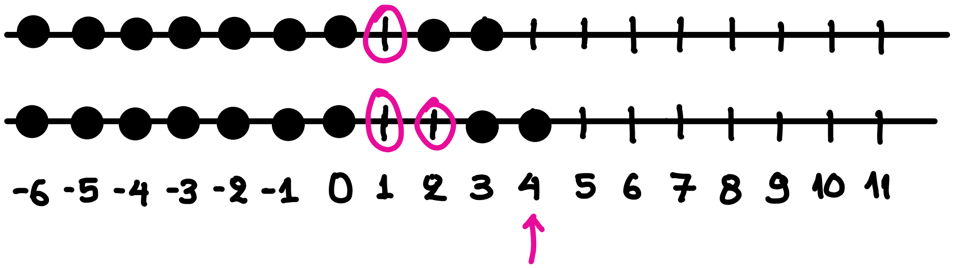
Def^n (Bessenrodt – Gramain – Olsson, 26/01/11)

$$HL^{BGO}(\lambda) = \{\{1, 1, 0, 2, 2, 1, 2, 2, 1, 3, 3, 2\}\}$$



$$HL^{CJ}(\lambda) = \{\{1, 4, 2, 5, 2, 1, -1, -4, 3, 2, 0, -3\}\}$$

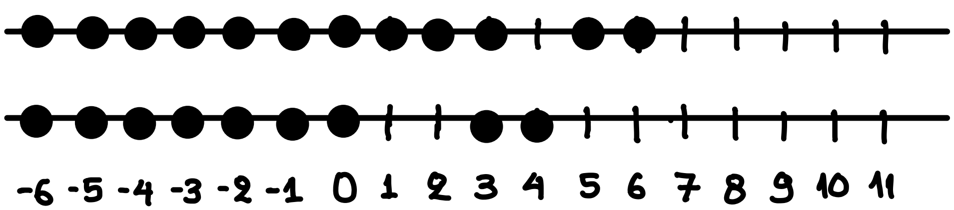
Ex. 



$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0 \}\}$

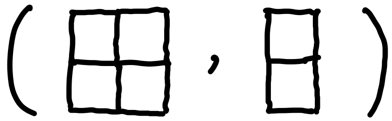
Def^n (Bessenrodt – Gramain – Olsson, 26/01/11)

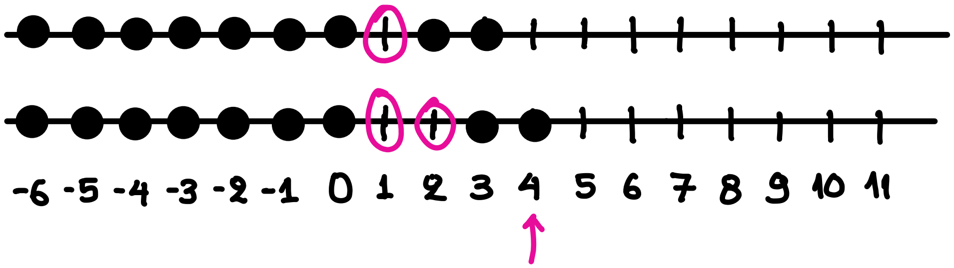
$HL^{BGO}(\lambda) = \{\{ 1, 1, 0, 2, 2, 1, 2, 2, 1, 3, 3, 2 \}\}$



$HL^{CJ}(\lambda) = \{\{ 1, 4, 2, 5, 2, 1, -1, -4, 3, 2, 0, -3 \}\}$

$HL^{BGO}(\lambda) = \{\{ 0, 1, 0, 2, 1, 4, 3, 1, 0, 5, 4, 2, 1, 0, 2, 1, 3, 2 \}\}$

Ex. 



$HL^{CJ}(\lambda) = \{\{ 1, 1, 2, 2, 2, 1, 2, -1, 3, 2, 3, 0 \}\}$

Def^n (Bessenrodt – Gramain – Olsson, 26/01/11)

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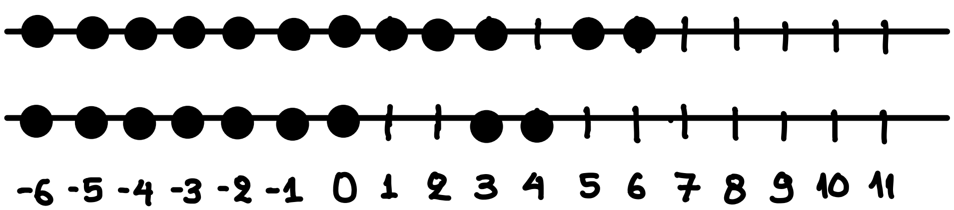

$HL^{CJ}(\lambda) = \{\{ 1, 4, 2, 5, 2, 1, -1, -4, 3, 2, 0, -3 \}\}$

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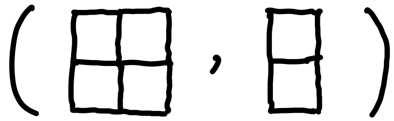
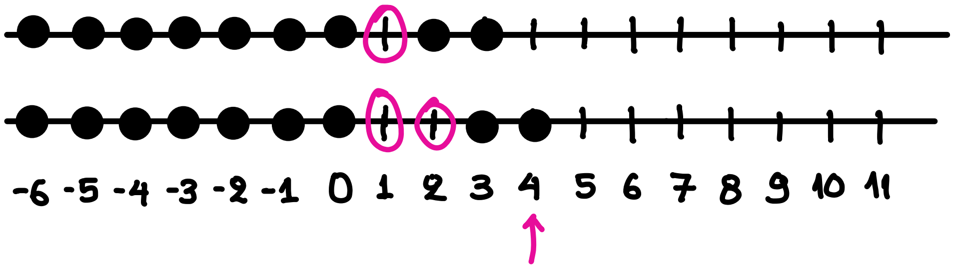
Ex. 



$$HL^{CJ}(\lambda) = \{\{1,1,2,2,2,1,2,-1,3,2,3,0\}\}$$

Def^n (Bessenrodt – Gramain – Olsson, 26/01/11)

$$HL^{BGO}(\lambda) = \{\{1,1,0,2,2,1,2,2,1,3,3,2\}\}$$

Thm (C. – Gramain – Jacon, '25)

There exists an injection $HL^{CJ}(\lambda) \longrightarrow HL^{BGO}(\lambda)$, $h \longmapsto |h|$

Thm (C.-Gramain-Jacon, '25)

Let λ be a partition of n and $e \in \mathbb{Z}_{>0}$. Then

$$\boxed{s_\lambda = s_{e\text{-core of }\lambda} \cdot s_{e\text{-quotient of }\lambda}}$$

Thm (C.–Gramain–Jacon, '25)

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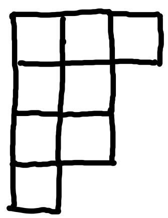
Ex $\lambda =$ 

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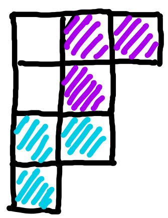
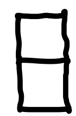
<u>Ex</u> $\lambda =$  3-core of $\lambda =$ 

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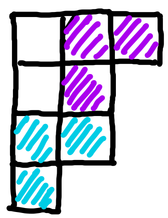
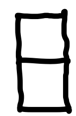
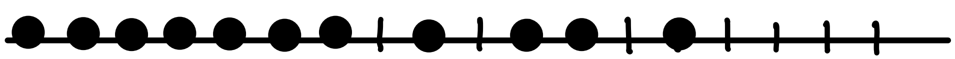
Ex $\lambda =$  3-core of $\lambda =$ 



Thm (C.-Gramain-Jacon, '25)

Let λ be a partition of n and $e \in \mathbb{Z}_{>0}$. Then

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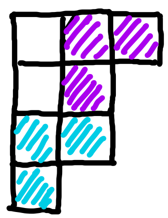
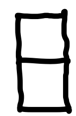
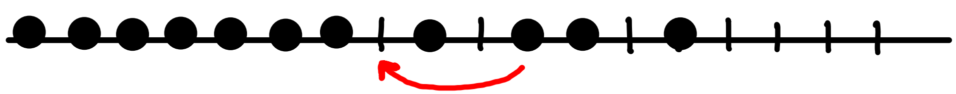
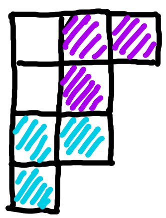
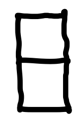
Ex $\lambda =$  3-core of λ = 

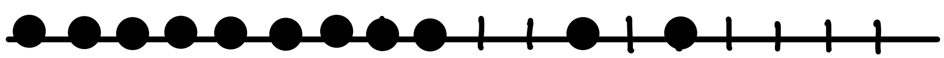


Thm (C.-Gramain-Jacon, '25)

Let λ be a partition of n and $e \in \mathbb{Z}_{>0}$. Then

$$\boxed{\, s_\lambda = s_{e\text{-core of } \lambda} \cdot s_{e\text{-quotient of } \lambda} \,}$$

Ex $\lambda =$  3-core of $\lambda =$ 



__Thm__ (C.-Gramain-Jacon, '25)

Let λ be a partition of n and $e \in \mathbb{Z}_{>0}$. Then

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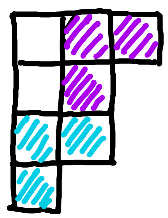
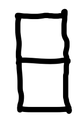
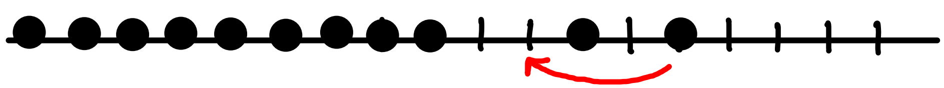
__Ex__ $\lambda =$  3-core of $\lambda =$ 



Thm (C.-Gramain-Jacon, '25)

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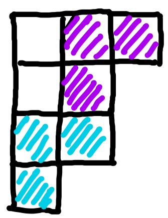
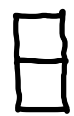
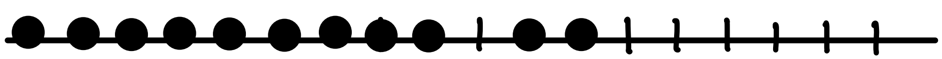
Ex $\lambda =$  3-core of $\lambda =$ 



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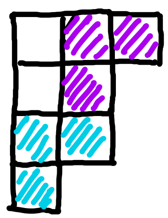
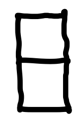
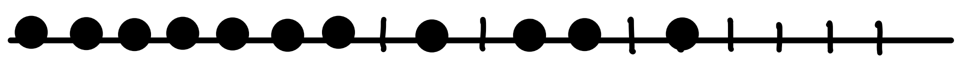
Ex $\lambda =$ 

3-core of $\lambda =$ 



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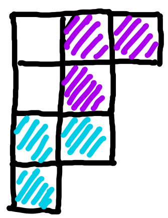
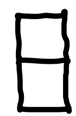
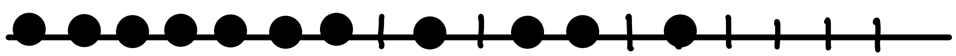
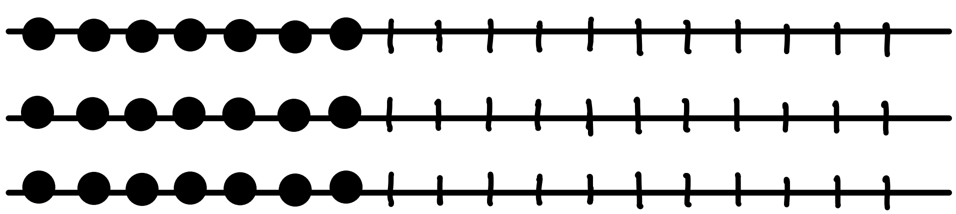
Ex $\lambda =$  3-core of $\lambda =$ 





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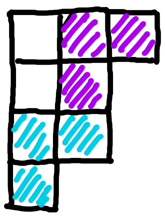
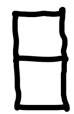
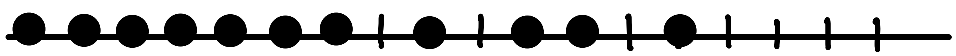
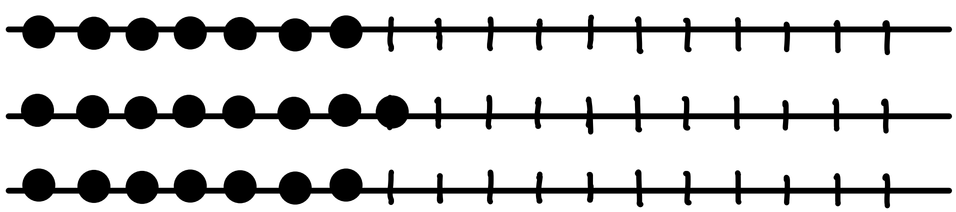
<u>Ex</u> $\lambda =$  3- core of $\lambda =$ 





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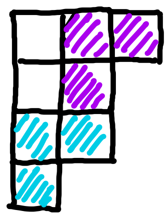
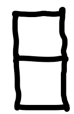
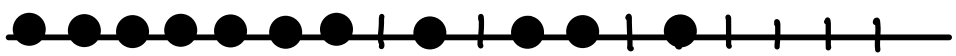
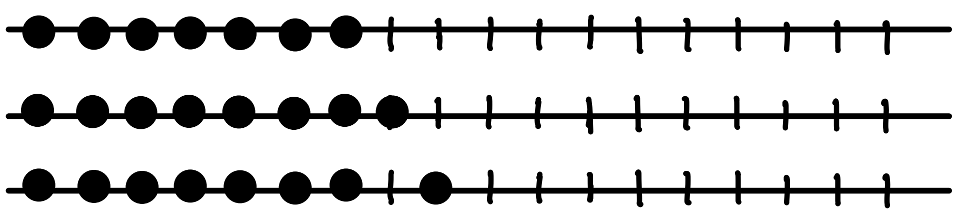
Ex $\lambda =$  3-core of $\lambda =$ 





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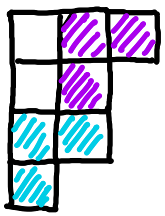
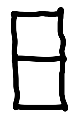
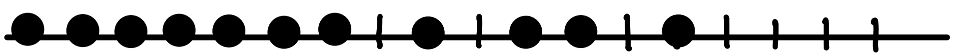
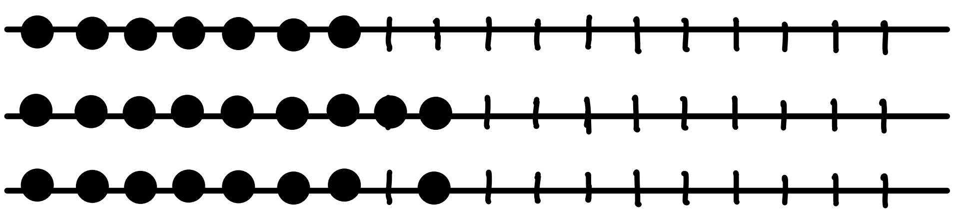
Ex $\lambda =$  3-core of $\lambda =$ 





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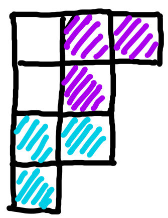
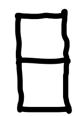
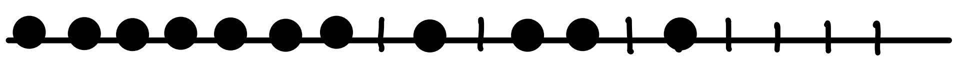
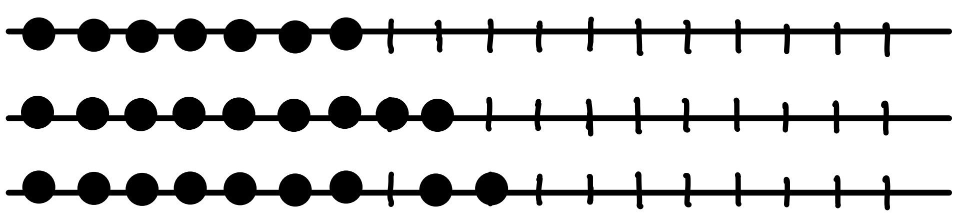
Ex $\lambda =$  3-core of $\lambda =$ 





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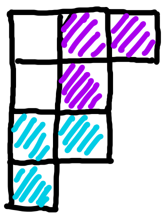
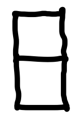
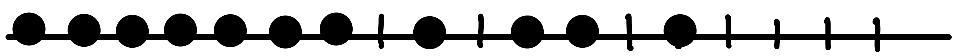
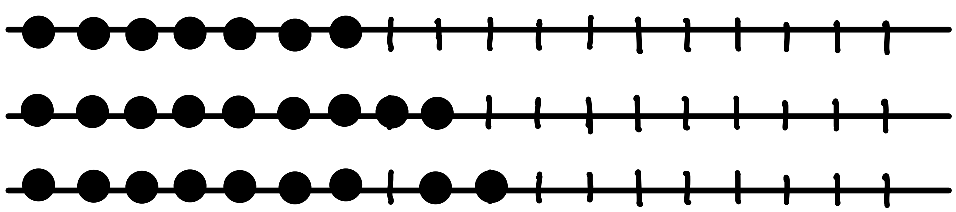
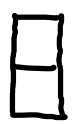
<u>Ex</u> $\lambda =$  3-core of $\lambda =$ 





3-quotient of $\lambda = ($  $, \emptyset, \emptyset)$

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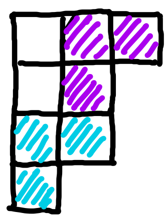
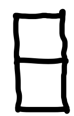
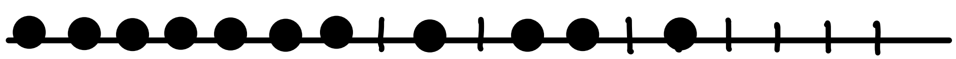
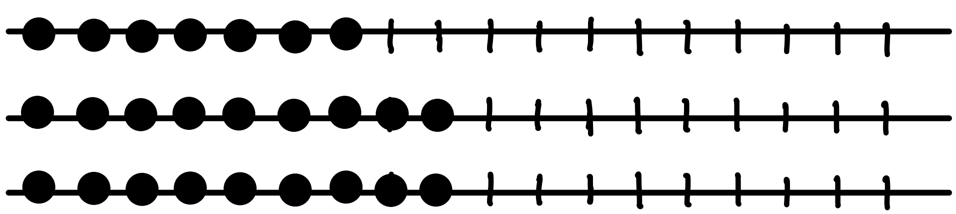
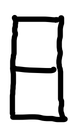
Ex $\lambda =$  3-core of $\lambda =$ 





3-quotient of $\lambda = \Big($  $, \emptyset, \emptyset \Big)$

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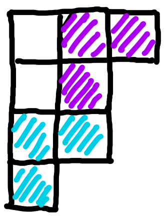
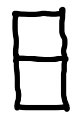
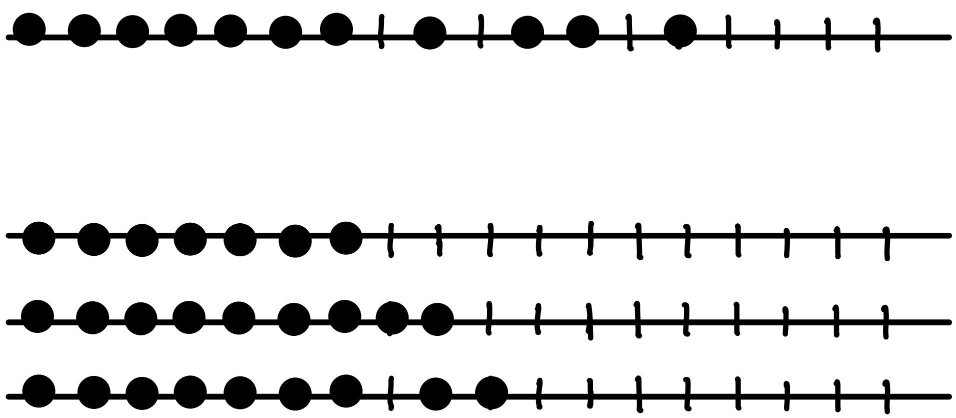
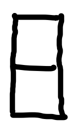
Ex $\lambda =$  3-core of $\lambda =$ 



3-quotient of $\lambda = ($  $, \emptyset, \emptyset)$

Merci beaucoup!