# Titles and abstracts

Closing conference of the ANR Cortipom

Le Croisic (France), June 2025

## Monday

**Anne Schilling (9h30-10h30):** Crystal skeletons: Combinatorics and axioms

Crystal skeletons were introduced by Maas-Gariépy in 2023 by contracting quasi-crystal components in a crystal graph. On the representation theoretic level, crystal skeletons model the expansion of Schur functions into Gessel's quasisymmetric functions. Motivated by questions of Schur positivity, we provide a combinatorial description of crystal skeletons, and prove many new properties, including a conjecture by Maas-Gariépy that crystal skeletons generalize dual equivalence graphs. We then present a new axiomatic approach to crystal skeletons. We give three versions of the axioms based on $GL_n$-branching, $S_n$-branching, and local axioms in analogy to the local Stembridge axioms for crystals based on novel commutation relations. This is joint work with Sarah Brauner, Sylvie Corteel and Zajj Daugherty (arXiv:2503.14782).

**Jean-Christophe Novelli (11h-12h):** Yang-Baxter elements, Jack polynomials, and beyond

The aim of this talk is to present apparently new conjectures about Macdonald and Jack polynomials. These conjectures are very elementary and can be checked easily with a computer algebra system.

The main ingredient is the definition of some Yang-Baxter elements (in the symmetric group algebra) that conjecturally give rise to Jack polynomials. Going through the ideas of other constructions, we shall generalize this to similar Yang-Baxter elements in the Hecke algebra that conjecturally give rise to Macdonald polynomials. Elementary conjectures will also be presented during this exploration.

Joint work with Jean-Yves Thibon.

**Elena Hoster (15h-15h30):** Chow polynomials and Friends

The Chow polynomial of a matroid is the Hilbert series of its Chow ring. This polynomial satisfies several combinatorial properties: it is palindromic, unimodal, and gamma-positive. Real-rootedness has been proven in special cases, but remains a conjecture for the general case. The Chow polynomial arises as an evaluation of the Poincaré-extended ab-index, a polynomial introduced by Dorpalen-Barry, Maglione, and Stump. This connection links Chow polynomials to a larger family of polynomials, including chain polynomials of posets and the coarse flag Hilbert-Poincaré series of matroids. Real-rootedness is a common conjecture within this family. In this talk, we will focus on the coarse flag Hilbert-Poincaré series and the Chow polynomial. For both, we will discuss special cases and open questions.

**Jad Abou-Yassin (15h30-16h):** A slice of Catalan combinatorics from the perspective of Coxeter sortable elements of the symmetric group

A large number of mathematical objects are enumerated by the famous Catalan numbers: triangulation of polygons, non-crossing partitions, number of ways to correctly parenthesize an expression, Dyck paths, binary trees, monotonic lattice paths,... I will introduce a less famous object that is also enumerated by the Catalan numbers: the Coxeter sortable elements of the symmetric group. They are specific permutations that are defined a priori in a way completely unrelated to the Catalan combinatorics.I will show explicit bijections from the Coxeter sortable elements to the non-crossing partitions and to binary search trees and

motivate their importance. If time is permitted, I will also discuss some generalizations to other Coxeter groups.

**James Parkinson (16:30-17h30):** Lusztig's asymptotic algebra for affine type A

Kazhdan-Lusztig theory plays a fundamental role in the representation theory of Coxeter groups, Hecke algebras, groups of Lie type, and algebraic groups. One of the most fascinating objects in the theory is the "asymptotic algebra" introduced by Lusztig in 1987. We will report on a new approach (joint with N. Chapelier, J. Guilhot, and E. Little) to construct the asymptotic algebra for affine type A, focusing on some of the main novelties of this approach, including the notion of a balanced system of cell modules, combinatorial formulae for induced representations, and an asymptotic version of Opdam's Plancherel Theorem.

**Inês Rodrigues (17h30-18h):** Crystals and quasi-crystals: applications to the combinatorics of quasi-symmetric functions

Crystal graphs are powerful combinatorial tools for working with the plactic monoid and symmetric functions. Quasi-crystal graphs are an analogous concept for the hypoplactic monoid and quasi-symmetric functions. We introduce a new combinatorial structure called quasi-array to explicitly describe an isomorphism between certain components of the quasi-crystal graph. As an application, we explore the interaction of fundamental quasi-symmetric functions and Schur functions.

# Tuesday

**Guillaume Chapuy (9h30-10h30):** On the scaling of random Tamari intervals and Schnyder woods of random triangulations (with an asymptotic D-finite trick)

We consider a Tamari interval of size $n$ (i.e., a pair of Dyck paths which are comparable for the Tamari relation) chosen uniformly at random. We show that the height of a uniformly chosen vertex on the upper or lower path scales as $n^{3/4}$, and has an explicit limit law. By the Bernardi-Bonichon bijection, this result also describes the height of points in the canonical Schnyder trees of a uniform random plane triangulation of size $n$. The exact solution of the model is based on polynomial equations with one and two catalytic variables. To prove the convergence from the exact solution, we use a version of moment pumping based on D-finiteness, which is essentially automatic and should apply to many other models. We are not sure to have seen this simple trick used before. Based on arXiv:2403.18719

**Mireille Bousquet-Mélou (11h-12h):** The ascent lattice on Dyck paths

Several posets defined on Dyck paths of length $2n$ have been studied, including in recent years: let us cite the Stanley lattice, the Tamari lattice and its greedy version... In particular, the enumeration of their intervals has revealed unexpected links with planar maps.

Here we consider a greedy version of Stanley's lattice, in which cover relations are obtained by exchanging a down step with the whole ascent that follows it. This order is connected to a more general lattice recently defined by Nadeau and Tewari, and it follows that it is also a lattice, the so-called "ascent lattice". One also considers the sub-posets induces on $m$-Dyck paths (all ascent lengths are multiples of $m$) and their mirrors (all descent lengths are multiples of $m$).

Counting intervals in these posets reveals links with walks plane confined to a cone. The associated generating function is algebraic for $m = 1$, for mysterious reasons, but not algebraic nor D-finite for $m > 1$. However, the numbers of intervals of mirrored $m$-Dyck paths occur in the OEIS, and one can establish a bijection between these intervals and congruence classes of the "Sylvester monoid" introduced in 2005 par Hivert, Novelli et Thibon.

This is a joint work with Jean-Luc Baril, Sergey Kirgizov (Université de Bourgogne, Dijon, F) and Mehdi Naima (Sorbonne Université, Paris, F)

**David Wahiche (15h-15h30):** Macdonald identities, affine Grassmannian elements and hook length formulae

The Nekrasov–Okounkov formula provides an expression of Fourier coefficients of powers of the Euler function as a sum of product of hook lengths. The aim of this talk is to these kind of formulas can be derived from a specialization of the Macdonald identity, also called Weyl–Kac denominator formula. The latter can be rewritten as a sum indexed by affine Grassmannian elements, where appears the atomic length introduced by Chapelier-Laget and Gerber. I then will try to build the dictionary in type A between these elements and some subsets of integer partitions. This is an introduction to a joint work with Cédric Lecouvey (https://arxiv.org/abs/2404.10532).

**Charlie Hérent (15h30-16h):** A discrete-time Matsumoto-Yor theorem

We study a random walk on the subgroup of lower triangular matrices of $SL_2$, with i.i.d. increments. We prove that the process of the lower corner of the random walk satisfies a Rogers-Pitman criterion to be a Markov chain if and only if the increments are distributed according to a Generalized Inverse Gaussian (GIG) law on their diagonals. For this, we prove a new characterization of these laws. We prove a discrete-time version of the Dufresne identity. We show how to recover the Matsumoto-Yor theorem by taking the continuous limit of the random walk.

**Marie Albenque (16h30-17h30):** Slice decomposition of hypermaps

Many bijections between maps and decorated trees have been developed in the last 20 years. In 2010, Jérémie Bouttier and Emmanuel Guitter introduced a new bijective paradigm for maps, called the "slice decomposition", which consists in cutting maps along some geodesic paths to produce some sort of canonical building blocks. This decomposition enables to obtain recursive decompositions, similar to the ones already available for decorated trees, but it also leads to new constructions and decompositions.

In my talk, I will present the extension of the slice decomposition to hypermaps (i.e maps in which faces can be properly coloured in two colours), which permits to derive bijective proofs for enumerative formulas obtained in the physics literature. This is a joint work with Jérémie Bouttier.

**Paul Thévenin (17h30-18h):** Character bounds, fixed-point-free permutations and random maps

We consider random walks on the Cayley graph of the symmetric group, where the generating set is a conjugacy class with no fixed point.

Using representation-theoretic tools, we refine bounds on characters from Larsen and Shalev, and use it to show that such random walks mix in two steps if the number of transpositions in the class is not too large, and in three steps otherwise.

An interesting application concerns the structure of random maps (that is, graphs on surfaces) obtained by gluing polygons by their edges.

This is joint work with Lucas Teyssier (UBC).

# Wednesday

**Neil O'Connell (9h30-10h30):** Discrete Whittaker processes

I will discuss a Markov chain on reverse plane partitions (of a given shape) which is closely related to the Toda lattice. This process has non-trivial Markovian projections and a unique entrance law starting from the reverse plane partition with all entries equal to plus infinity. I will also outline some connections with imaginary exponential functionals of Brownian motion, a random polymer model with purely imaginary disorder, interacting corner growth processes and discrete delta-Bose gas, and hitting probabilities for some low rank examples.

**Maria Chlouveraki (11h-12h):** Generalizing hook lengths for partitions

Representation theory and algebraic combinatorics have always been interconnected, since the irreducible representations of the symmetric group are labelled by partitions. The dimensions of said representations are given by the hook length formula. In this talk, we will discuss about how we can generalize the notion of hook length to multipartitions so that we get useful information about the representation theory of objects that multipartitions label, namely the irreducible representations of other Coxeter groups, complex reflection groups and Ariki-Koike algebras. We deduce some surprising connections between combinatorial notions attached to these representations. The talk is based on joint work with Nicolas Jacon and Jean-Baptiste Gramain.

# Thursday

**Cesar Cuenca (9h30-10h30):** Random partitions at high temperature

By using Fourier transforms based on the Jack symmetric polynomials, we study discrete particle ensembles $x_1 > x_2 > ... > x_N$ with the inverse temperature $\beta > 0$ in the regime where beta tends to zero, as the number of particles tends to infinity. We prove the LLN and characterize the limiting measure in two ways: as the unique solution to a moment problem and a functional equation for its characteristic function. As an example, we briefly comment on the fixed-time distribution of certain Markov chain of nonintersecting particles. This talk is based on joint work with Maciej Dołęga.

**Christian Krattenthaler (11h-12h):** Positive $m$-divisible non-crossing partitions and their cyclic sieving

Buan, Reiten and Thomas (implicitly) defined positive $m$-divisible non-crossing partitions, by setting up a bijection between the facets of the generalized cluster complex of Fomin and Reading and $m$-divisible non-crossing partitions, positive clusters corresponding to positive $m$-divisible non-crossing partitions. We embark on a finer enumerative study of these combinatorial objects associated with finite reflection groups. In particular, we define a cyclic action on them, which — together with the "obvious" $q$-analogue of positive Fuß–Catalan numbers — satisfies the cyclic sieving phenomenon of Reiner, Stanton and White. Our proof is a — lengthy — case-by-case verification. Crucial in the proof in the classical types are combinatorial realisations of the positive $m$-divisible non-crossing as certain classical $m$-divisible non-crossing partitions on the one hand and the combinatorial description of the action as some kind of pseudo-rotation on the other hand.

This is joint work with Christian Stump.

**Thibaut Lemoine (15h-15h30):** Asymptotic representation theory of unitary groups and application to Yang-Mills theory

In this talk, I will discuss several results about the representation theory of large unitary groups. In particular, I will describe how the highest weights of $\mathrm{U}(N)$ can be interpreted as couplings of integer partitions, and what it tells us about the dimension and the Casimir number of the associated irreducible representations. I will then provide a few applications to two-dimensional Yang–Mills theory.

**Harriett Walsh (15h30-16h):** Random growth in half space

I will talk about a model of two dimensional random growth (namely, polynuclear growth) which can be translated into a probability law on integer partitions (by way of the RSK algorithm). We can find exact expressions for statistics of this model with algebraic tools, and compute fine asymptotics. I will focus on the model in half space with external sources driving growth at the edges. The limiting distribution of interface fluctuations in this model interpolates between different universal Tracy—Widom distributions from random matrix theory, and encodes solutions of the Painlevé II differential equation. At one point it matches a half-space version of the Baik—Rains distribution found by Barraquand, Krajenbrink and Le Doussal using methods from physics. Our approach uses connections between symmetric functions, matrix

integrals, and Hankel determinants, plus a Riemann–Hilbert problem. Based on joint work with Mattia Cafasso, Alessandra Occelli and Daniel Ofner.

**Filippo Colomo (16h30-17h30):** The Tangent Method: an introduction

The Tangent Method [FC, A Sportiello, 2016] is a recipe for the exact evaluation of arctic curves or limit shapes in a variety of models. In the case of non-intersecting lattice paths the method is very simple and intuitive, and its validity may be proven rigorously. Extension of the method to dimer models, plane partitions, or, more generally, free-fermionic models, is straightforward.

Extension of the method to further models, beyond free-fermionic (or determinantal) ones, although heuristic, has proven to be very effective. Indeed, the obtained results are in full agreement with numerics, as well as with alternative analytic derivations, when available. Occasionally the validity of the method has also been proven rigorously, on a case-by-case basis.

Here we provide an elementary introduction to the method, together with a few pedagogical examples and applications.

Time permitting, we shall also briefly present a recent result [FC, AG Pronko, 2024], according to which the fluctuations of the shape of large Alternating Sign Matrices are governed by GUE Tracy-Widom distribution.

**Francesco Casini (17h30-18h):** Duality and integrability for the multi-species stirring process with open boundaries

To construct a model for non-equilibrium statistical mechanics, the system is typically brought into contact with two thermodynamic baths, referred to as boundary reservoirs. These reservoirs impose their own density of particles at the system's boundary, generating a current. In the long-time limit, a non-equilibrium steady state sets in, characterized by a stationary value of the current.

Currently, there is a growing interest for multi-component systems, i.e. models where many different species of particle (sometimes called colours) are present.

This presentation focuses on the boundary-driven multi-species stirring process. This process is a natural extension of the symmetric exclusion process (SEP) when multiple species of particles are considered. Its dynamics involve the exchange of positions between a particle and a hole or between two colours of particles, both occurring at a rate of 1. In addition to this "bulk" dynamics, the system is put in contact with boundary reservoirs that inject, remove and exchange type of particles.

After describing the process's generator using a suitable representation of the $\mathfrak{gl}(N)$ Lie algebra, we establish the existence of an absorbing dual process defined on an enlarged chain, in which each boundary reservoir is replaced by an absorbing extra-site. This dual process shares the same dynamics in the bulk, but the extra-sites absorb particles voiding the graph in the long time horizon.

We combine absorbing duality and the matrix product ansatz to derive closed expressions for the non-equilibrium steady-state multi-point correlations of the process. Consequently, we formulate exact expressions for the non-equilibrium steady state.

Finally, we discuss some extensions to the non-integrable chain. This presentation is based on the recent joint work with Rouven Frassek and Cristian Giardinà "Duality for the multispecies stirring process with open boundaries", 2024 *J. Phys. A: Math. Theor. 57 295001.*

# Friday

**Olga Azenhas (9h30-10h30):** Keys, virtual keys and applications

Demazure atoms are core components in the description of Demazure crystals. It is natural to ask to which Demazure atom a given vertex of a crystal belongs. The answer to this question is provided by the right key map (respectively left key for opposite Demazure atoms). The effective computation of the key map has therefore captured interest across different areas.

Demazure modules were originally described as the space of global sections of a suitable line bundle on a Schubert variety. This description exhibits the natural correspondence between Schubert varieties and

Demazure modules and henceforth their combinatorial skeletons, Demazure crystals or Demazure crystal atoms. In joint work, with Gobet-Lecouvey, Demazure pieces have been shown to describe RSK correspondence when rectangular nonnegative matrices are replaced with certain general staircase-shaped matrices which permitted to study the LPP model on those matrices. More recently, Feigin-Khoroshkin-Makedonskyi have generalized the Cartan type $A$ non-symmetric Cauchy identities by Lascoux, Azenhas-Emami, and Azenhas-Gobet-Lecouvey to arbitrary staircase-shaped matrices.

Originally, the relevance of keys stems from standard monomial theory. The Lascoux–Schützenberger right and left keys computed via jeu de taquin on semistandard Young tableux were designed to encapsulate the Lakshmibai–Seshadri minimal respectively maximal defining chains in standard monomial theory. Since then the computation of keys has been generalized either in Cartan type or using a specific crystal model.

We present a new technique for computing the key map and the Schützenberger–Lusztig involution using virtualization of crystals. Kashiwara introduced a method for embedding highest weight $g_X$-crystals inside highest weight $g_Y$-crystals, where $g_D$ is the complex simple Lie algebra associated to the Dynkin diagram $D$, provided the Dynkin diagram $X$ can be obtained from the Dynkin diagram $Y$ via a Dynkin diagram folding. This shows that key map on crystals can be reduced to the simply-laced types. The results are type-independent and crystal model-independent. This is based on joint work with González-Huang-Torres.

**Paolo Papi (11h-12h):** The role of affine Weyl groups in some problems of combinatorics and representation theory

I will discuss some instances of the emergence of affine Weyl groups as a basic tool to solve problems in representation theory and combinatorics. I will consider on some detail the study of B-orbits on abelian ideals (joint works with Gandini, Maffei and Mosender Frajria) and an analog of Panyushev's rootlet theory for infinitesimal symmetric spaces. The latter topic has been investigated in the PhD thesis of F. Stara and it is the subject of ongoing research.